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Ambivalence in decision making: An eye tracking study

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ABSTRACT

An intuition of ambivalence in cognition is particularly strong for complex decisions, for which the merits and demerits of different options are roughly equal but hard to compare. We examined information search in an experimental paradigm which tasked participants with an ambivalent question, while monitoring attentional dynamics concerning the information relevant to each option in different Areas of Interest (AOIs). We developed two dynamical models for describing eye tracking curves, for each response separately. The models incorporated a drift mechanism towards the various options, as in standard drift diffusion theory. In addition, they included a mechanism for intrinsic oscillation, which competed with the drift process and undermined eventual stabilization of the dynamics. The two models varied in the range of drift processes postulated. Higher support was observed for the simpler model, which only included drifts from an uncertainty state to either of two certainty states. In addition, model parameters could be weakly related to the eventual decision, complementing our knowledge of the way eye tracking structure relates to decision (notably the gaze cascade effect).

1. Ambivalence in decision making: An eye tracking study

We have all had an experience of ambivalence in decision making, especially in cases of ill-matched, incommensurable, vaguely specified, options. For example, the two options for a Friday evening, 'going to the movies' vs. 'ordering takeaway' arguably have no matching dimensions along which they can be compared. An experience of ambivalence would then be one of vacillation between the two options, so that a person is close to preferring one vs. the other on a number of occasions, before gradually converging to the eventual choice. This introspection for decision making dynamics contrasts sharply with the usual assumption that evidence accumulation is fairly monotonic (even if still stochastic) towards the dominant choice, until a decision is reached, as in diffusion theory (e.g., Ratcliff & Smith, 2015) or other models (e.g., Jekel, Glöckner, & Bröder, 2018). Diffusion models have been enormously successful and clearly there are plenty of situations where decision dynamics are described extremely well by such models. With the present work,

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we want to explore whether there is plausibility in the notion that, sometimes, decision dynamics conform to a pattern of oscillation/fluctuation, followed by stabilization, whereby we assume that oscillation or fluctuation is one marker of ambivalence (note, we employ the terms oscillation and fluctuation interchangeably and, for brevity, we will use just the former term henceforth; for some indirect evidence for oscillations in decision making see Brehm & Wicklund, 1970, and Walster, 1964).

Unfortunately, there are challenges in probing decision dynamics directly. There is extensive evidence that sometimes decisions can alter the relevant mental states and so influence any further decision making (Schwarz, 2007; Sharot, Velasquez, & Dolan, 2010; White, Pothos, & Busemeyer, 2014; White, Pothos, & Jarrett, 2020; Yearsley & Pothos, 2016). For example, a decision might reveal insights or perspectives which influence subsequent decisions (Schwarz, 2007) or it might 'disturb' the mental state in certain ways (e.g., White et al., 2014). Kvam, Busemeyer, and Pleskac (2021) circumvented this problem by employing a paradigm whereby different participants were asked for preference ratings at different times, through a prolonged decision task. By averaging across participants, it was possible to infer the dynamics of preference for all decision points. Kvam et al. (2021) approach has many strengths, but it is clearly also desirable to study dynamics across the entire decision period within participants. Accordingly, a process tracing approach is needed which does not require overt judgments (Norman & Schulte-Mecklenbeck, 2009; Payne, Bettman, & Johnson, 1993).

A proxy for decision dynamics is the attentional dynamics for the relevant stimuli, up to the point of the eventual decision (Glaholt & Reingold, 2011; Orquin & Mueller Loose, 2013). Attentional dynamics can be continuously and robustly measured using eye tracking (Deubel & Schneider, 1996; Hoffman & Subramaniam, 1995; Kowler, Anderson, Dosher, & Blaser, 1995; Theeuwes, Belopolsky, & Olivers, 2009; Rizzolatti et al., 1987). Eye-tracking has been successfully used in many studies on decision making to investigate the underlying cognitive processes and dynamics (e.g., Fiedler & Glöckner, 2012; Glöckner & Herbold, 2011; Gluth, Kern, Kortmann, & Vitali, 2020; Krajbich & Rangel, 2011; Lohse & Johnson, 1996; Russo, 1978; Russo & Leclerc, 1994). So, in the present work we focus on attentional variables from eye tracking, concerning fixations corresponding to a choice between two unmatched, multi-attribute options.

The question of the putative link between attentional and decisional dynamics is a fascinating one. It is well known that attention can affect motivation for action (Suri & Gross, 2015) and, in certain cases, attentional biases can predict corresponding behaviors. For example, attentional biases for alcohol-related information can predict changes in days of alcohol drinking in a sample of excessive drinkers (Cox, Pothos, & Hosier, 2007) and attentional biases for unhealthy foods changes in the Body Mass Index (Calitri, Pothos, Tapper, Brunstrom, & Rogers, 2010). Rich theory has developed for how such attentional biases might lead to higher alcohol consumption or eating (Cox, Fadardi, & Pothos, 2006).

Additionally, there have been several empirical findings which support a putative link between attentional and decisional dynamics. First, the gaze cascade effect is the observation that there is increasing attentional focus for the eventually chosen option, assuming that all options are visually, concurrently presented (Fiedler & Glöckner, 2012; Glaholt & Reingold, 2009a,b; Krajbich, Armel, & Rangel, 2010; Shimojo, Simion, Shimojo, & Scheier, 2003; Simion & Shimojo, 2006; 2007; Stewart, Gächter, Noguchi, & Mullett, 2016). The gaze cascade effect appears robust across a range of situations, varying the type of choice, the visual display leading to a decision (Simion & Shimojo, 2007), and the complexity of the decision (e.g., Schotter, Berry, McKenzie, & Rayner, 2010). Second, ruling out alternatives during a decision task reduces or eliminates attention towards these alternatives (Scholz, Krems, & Jahn, 2017). Third, there is evidence that the more one looks at an alternative, the more evidence for this alternative accumulates and the higher the chance that the alternative will be eventually selected (this is the mere exposure effect, Armel, Beaumel, & Rangel, 2008; Mullett & Stewart, 2016). Some evidence accumulation decision models have incorporated such a mechanism of linking evidence accumulation towards an option with attention towards the option (Krajbich et al., 2012; Noguchi & Stewart, 2018). Finally, it appears that during the course of a decision, different options will be attended to differently, depending on evidence accumulation for these options (Gluth et al., 2020).

We employed a decision situation designed to embody ambivalence, so that participants had to consider their preference for adopting an option vs. not, on the basis of several, poorly matched pros vs. cons for the option. We used eye tracking to record fixations towards the Area of Interest (AOI) with the information for adopting the option (and a corresponding preference button) vs. the AOI for rejecting the option. There was an expectation that there would be 'oscillations' in the attentional dynamics for adopting vs. rejecting the option, that is, a repeating pattern of focus towards one AOI, followed by focus towards the other, and back again. Also, the gaze cascade effect and related findings would make us expect that such oscillations would gradually quench themselves and attention be gradually concentrated towards the eventually chosen option. Given these expectations, we utilized quantum theory to construct two nested models for attentional dynamics, for an ambivalent decision task.

Quantum mechanics is a theory of physics, but it also embodies a theory for assigning probability to events – what we call quantum theory (e.g., Hughes, 1989; Isham, 1989). Quantum theory is potentially applicable in any situation where there is a need to formalize uncertainty, including in psychology (Busemeyer & Bruza, 2011; Khrennikov, 2010; Pothos & Busemeyer, 2013, 2022). In this work, we employ the apparatus for dynamical evolution of probabilities from quantum theory. In quantum theory (and more generally), there is a key distinction between closed or isolated systems (which do not interact with their environment) and open systems (for which we have to take into account interactions with their environment). For closed quantum systems, the dynamics are typically characterized by indefinite oscillation (Bagarello, Basieva, & Khrennikov, 2018). For open quantum systems, the dynamics typically reflect gradually quenched oscillations and eventual stabilization. A priori, given findings such as the gaze cascade effect, this pattern of gradually quenched oscillations struck us as a plausible way to approach the problem of modelling attentional dynamics in an ambivalent decision task. Mathematically, in open quantum systems (OQS) dynamics there is a part which biases towards indefinite oscillation (the Schrödinger part, see below) and a part which biases towards stabilization (the Lindblad term). Thus, in the quantum theory of OQS, we have a dynamical framework embodying two competing processes, one of indefinite ambivalence vs. one of gradual stabilization. Note, our use of quantum theory assumes a fully classical brain; all quantum processes are epiphenomenal and concern computational-level principles which are offered as descriptors of behavior (Yearsley & Pothos, 2014).

Note, the use of quantum theory further supports the notion that we cannot study decisional dynamics by asking the same participant multiple questions during her decision (cf. Kvam et al., 2021). If we model mental states with quantum states, then we have to take into account that a decision typically changes quantum-like states, so that the new state coincides with the outcome of the decision (White et al., 2014, 2020).

Directly modeling attentional dynamics has been rare. Tatler, Brockmole, and Carpenter (2017) provided a detailed model for predicting saccades in a step-by-step manner, when processing an image. Based on a combination of several image features, their model could predict the timing and location of the next saccade. However, a decision theorist might be more interested in attentional dynamics towards different areas of interest (AOIs) across time. Models for such dynamics have rarely been proposed. Some theories rely on an extension of evidence accumulation models to describe interactive activation (Glöckner & Herbold, 2011; Jekel et al., 2018). These models propose that the emerging attractiveness of an option (its activation) increases the likelihood of searching information for that option (and the related AOIs). This attraction search effect has received strong empirical support (Jekel et al., 2018; see also Glöckner & Herbold, 2011), but corresponding models may be less well suited to ambivalent decisions, for which we might expect some oscillation of attentional focus between options.

Alternatively, it could be argued that the current dominant frameworks of sequential sampling models for decision dynamics, random walk or diffusion models (Ratcliff & Smith, 2015), could be profitably employed in the case of attentional (e.g., eye tracking) dynamics as well.

Diffusion models typically concern two-choice tasks. At each step in the decision process, it is assumed that some evidence is sampled. All the relevant information that could be sampled is summarized in (in the simplest case) a single parameter, called the drift rate. The drift rate provides a stochastic expectation of which of the two options will be favored in each step; it does not have to be fixed and variability in the drift rate can conform to different distributions. Across time, once there is enough evidence for a particular option to cross a pre-determined threshold, the decision concludes. Diffusion models have been incredibly successful, typically predicting both choice and error distributions (Ratcliff & Smith, 2015).

As an example, in Decision Field Theory (DFT; Busemeyer & Townsend, 1993), at each time step, two options are evaluated using a single attribute. A stochastic sampling process determines momentary evaluations and so shifts in preference towards one or the other option. Diederich (2003) notes that an initial bias towards the less favored option can produce a preference reversal, in the sense that the evidence accumulation process eventually supports the initially disfavored option. Note, such a process would at most lead to a single preference reversal, not multiple ones. Multivariate DFT (MDFT) assumes stimuli made of several attributes and incorporates an attention process that switches attention from attribute to attribute, based on some (usually fixed) transition probabilities. At each step, relative preference for the two choices is adjusted based on the currently activated attribute (Diederich, 2003; Johnson & Busemeyer, 2005). Additionally, in MDFT there is a mechanism of lateral inhibition, so that evidence can be inhibited as a result of accumulated evidence for another alternative.

Beyond the MDFT, there are several other multi-attribute diffusion models. For example, in the multiattribute ballistic accumulator model (MLBA; Brown & Heathcote, 2008; Trueblood et al., 2014) evidence accumulation depends on an overall drift rate, which is based on weighted differences along all attributes. The MLBA lacks a mechanism of lateral inhibition. Multivariate decision by sampling (MDbS; Noguchi & Stewart, 2018) works with attribute-specific, ordinal comparisons, so that attentional focus for particular attributes depends on a similarity function between attributes. The Attentional Drift-Diffusion Model (aDDM; Krajbich et al., 2012) computes a time-dependent decision variable, that changes over time, and eventually triggers a decision when a threshold is crossed. This decision variable changes at different rates, depending on both the characteristics of the available options and attentional fixations. Fixations alter drift rates, longer fixations towards one option make eventual preference for that option more likely (if the option has overall positive value), and the last fixation has a large impact on choice (this last assumption provides an interesting perspective on attentional biases in psychopathology, e.g., Cox et al., 2007). As with some other models, the aDDM assumes that preference for each choice is summarized with a single, subjective value.

The theoretical landscape of multi-attribute diffusion models is complex (Fuss & Navarro, 2013). Presently, we are concerned with two issues. The first issue is whether such models can capture a putative pattern of a 'few' gradually quenched oscillations, followed by stabilizations. Note, in eye tracking, researchers often employ the notion of a transition, which is defined as a change of attentional focus from one AOI to the other. We define an oscillation as a change in emphasis, to favor one AOI vs. another, i.e., whether fixations are predominantly in one AOI vs. the other. In a dynamics curve, oscillations would correspond to the number of peaks and troughs. So, even if focus predominantly favors one AOI, there could still be oscillations, in terms of whether we are more strongly vs. weakly focused on that AOI. We contend that in a case of ambivalent decision making there would be cases of at least some oscillations (more than two) in the dynamics curve, which are gradually quenched as we approach a decision (as per the gaze cascade effect, Shimojo et al., 2003).

To return to this first issue, can such putative oscillatory behavior be accommodated within multi-attribute diffusion models? We think not, because such models at heart embody a trend from uncertainty to certainty. Some reversals in preference might emerge, because drift rates are directly or indirectly a function of the attributes of the compared alternatives (even when summary drift rates are assumed, in each step drift rates could be computed from a random sample of the available attributes). However, such models typically do not have a natural way to quench a pattern of early oscillations. One apparent exception is the aDDM, which predicts evidence accumulation towards the option that is currently fixated to; respective preference reversals would then be dependent on the duration of fixations. Still, aDDM does not model the emergence of fixations, but uses the empirically observed fixations to model the decision process (a similar point applies to Gluth et al., 2020). Our main purpose is to present a framework, which shares some commonalities with diffusion models, notably drift rates, but otherwise provides a departure from such models with various distinctive features (including the capacity for oscillations).

Notwithstanding our expectation that multi-attribute diffusion models are poorly suited to the modeling of gradually quenched oscillations, the second issue is whether such models could in principle be fitted to attentional dynamics. Technically, to fit the curve from a diffusion model to the eye tracking curve from a participant, we would need to select diffusion parameters and then simulate lots of diffusion processes, to obtain an 'average' prediction for the curve of decisional propensities with time, which could be matched to empirical curves – and repeat this process for several different parameters (these could be identified e.g. via grid search; e.g., see Kohl et al., 2020). Computationally, this would be very demanding, since for each set of diffusion parameters we would need to run a separate simulation, to match an empirical curve, separately for each decision in our data set – as we will shortly see, we collected data for 45 (participants) $\times$ 3 (stories) $\times$ 2 (decision stages) = 270 decisions.

Another problem is that, theoretically, it is suspect that this would be correct in the first place. A curve of decisional propensities from a diffusion model is intended to eventually reveal the participant's response. This is not so for the eye tracking curves. Therefore, the 'best fit' curve from a diffusion model, that is, the curve which best matches a particular eye tracking curve, might indicate a response different to that the participant made (and at a time different from the actual response time). By contrast, eye tracking curves are not constrained in this way. The gaze cascade effect (that we observed in the present data) offers some correlation between eventual response and the curve at the time point when a choice is made, but this addresses in a very partial way this concern.

Overall, we think there is a need for a diffusion-style model, which can be used to directly describe attentional (or even decisional) dynamics, especially in situations of ambivalence for which we might expect at least a few oscillations.

1.1. Open quantum systems (OQS) dynamics

Both quantum theory and classical Bayesian theory involve rules for how to assign probabilities to events and how probabilities might evolve in time (with Schrödinger or Lindblad equation in the former case and the Kolmogorov forward equation in the latter case). Quantum theory and Bayesian theory are based on different axioms. Cognitive psychologists have considered quantum cognitive models in cases where human behavior appears at odds with Bayesian principles (Busemeyer & Bruza, 2011; Haven & Khrennikov, 2013; Khrennikov, 2010; Pothos & Busemeyer, 2013). We focus discussion on dynamics. The Kolmogorov forward equation for the classical approach provides a picture of evidence accumulation similar to that in diffusion models – the typical pattern is one of gradual increase of preference for the dominant options.

In quantum theory, there are two kinds of dynamics, depending on whether a system can be assumed to be isolated (closed) or interacting with its environment. In cognitive models, the system might be the information and thoughts for a particular task at hand (we question this approach just below). The relevant dynamics for a closed quantum system S is governed by the Schrödinger equation. The environment, E, relevant to system S, corresponds to information and thoughts a person can have in general and beyond what is specific about the task at hand. In quantum theory, the interaction between E and S assumes that both E and S have a quantum description and the pair C=(S,E) is treated as a compound quantum system. Under some conditions/ approximations, we can extract the state of the subsystem S from the compound system C, and the resulting dynamics is described by the Lindblad equation, that is, the quantum master equation. The term OQS refers to systems S interacting with their environment E and the resulting theory is the most general theory of interaction of a system with its environment.

Isolated systems governed by Schrödinger dynamics typically retain any quantum character indefinitely. A typical characteristic of such dynamics is indefinite oscillation. Schrödinger dynamics can be specified in a way closely analogous to Kolmogorov dynamics and has been fairly frequently employed in cognitive modelling. Because oscillation means that there is a back-and-forth between preference for one option vs. preference for the other, researchers employing such dynamics have typically assumed that a decision is forced at a certain time point (e.g., Pothos & Busemeyer, 2009). Open systems, that is systems interacting with their environment, gradually (in asymptotic time) lose all quantum character. Oscillations are gradually quenched, and the dynamics of the system settle to a constant level. This process is called stabilization or decoherence. That is, OQS dynamics are typically characterized by oscillation followed by stabilization, as the Lindblad part in the Lindblad equation eventually dominates the Schrödinger part. The speed of decoherence depends on the strength of the interaction with the environment. Applications of OQS dynamics in psychology have been limited (a fairly comprehensive list is Asano, Ohya, Tanaka, Basieva, & Khrennikov, 2011; Asano, Ohya, Tanaka, Khrennikov, & Basieva, 2011; Broekaert, Basieva, Blasiak, & Pothos, 2017; Kvam et al., 2021).

The theory of OQS was originally developed for microscopic physical systems, interacting with their environment. It is a reasonable question how well this picture can translate to the study of the mind. A physical target system with few degrees of freedom is analogous to the specific task a participant is called to complete. The relevant environment of the target system (many degrees of freedom) is analogous to the general knowledge of the participant. The more we can assume that a target physical system is isolated from its environment, the more Schrödinger dynamics can be used to describe it. Analogously, if we can assume that a person is able to fully focus on the information for a task at hand (S), without influence from his general knowledge and beliefs, Schrödinger dynamics applies and there would typically be indefinite oscillation. By contrast, Lindblad dynamics applies when the decision process is influenced by the person's general knowledge and beliefs (E); this interaction eventually leads to stabilization of the dynamics at a level which corresponds to the person's decision. That is, the interaction between the information for a particular problem and the person's general knowledge and beliefs is what eventually helps the person make up his mind. When this is the case, quantum character in S is lost and decoherence will occur. Note, in general the more numerous degrees of freedom of E will 'match' the fewer degrees of freedom of S, but unmatched degrees of freedom in S, if there are any, will continue contributing to indefinite oscillation. Note also that it is assumed that while the effect of E on S is so strong that it determines the final stable state of S, the effect of S on E is negligible. It is possible that these ideas could be developed to formalize Fodor's (1983) influential proposal that higher cognitive processes cannot be studied in isolation, irrespective of the general knowledge and beliefs of individual persons, though at this point such a proposal is speculative.

Motivated from the technical point that OQS dynamics typically involves a characteristic pattern of (fewer or more) oscillations followed by stabilization, we sought to develop two nested OQS models for attentional dynamics in an ambivalent decision task. Before proceeding with technical details, we outline the main differences between the OQS models and diffusion models.

First, an OQS model involves no thresholds and no discrete decision point. Rather, an eventual decision is inferred from the way dynamics eventually stabilize and the decision time could be any point in the period after which oscillations have been 'sufficiently' quenched. Inferring decisions from stabilization is not uncommon in the literature and, for example, is assumed by various models such as those of Glöckner and Betsch (2008), McClelland, Mirman, Bolger, and Khaitan (2014), or Thagard (1989). Some of the findings supporting these models cannot easily be reconciled with classic decision-to-threshold models (e.g., coherence effects; Holyoak & Simon, 1999; or the attraction search effect, Jekel et al., 2018). Specifically, coherence effects (a.k.a. predecisional information distortions) refer to the well-established phenomenon that the subjective perception of cues, values and attributes is changed during the decision process (see DeKay, 2015, for a recent review). Since decision-to-threshold models take these pieces of information only as input to accumulate overall values for options (or a difference in value), such changes in the input are not predicted and cannot be explained. The attraction search effect describes the phenomenon that search is not stationary but changes dynamically during the decision process: participants' tendency to search information concerning one option increases with the current attractiveness of this option. Standard decision-to-threshold models, in contrast, typically assume a stationary process of information sampling, which cannot account for such dynamic effects (but see Gluth et al., 2020).

Other findings particularly from neuroscience, however, also support decision-to-threshold models (see Gold & Shadlen, 2007, for a review). It seems, therefore, that there is evidence in the literature for both kinds of processes. A stabilization account might be more applicable in more complex, longer, and more deliberate decision processes (e.g., in legal reasoning or the ambivalent decision paradigms we will shortly describe), since it avoids the possibility that a threshold is accidentally crossed when the system vacillates between various complex options or interpretations. For example, consider an indefinite sinusoidal pattern; if a threshold is crossed close to the first oscillation, then the subsequent structure is missed. Clearly, more complex formalisms (e.g., a decision-to-threshold model with collapsing boundaries) would not fall foul of such potential problems. Either way, this is not an issue which can be settled without further work, though we contend that the OQS choice is reasonable for the present case.

Second, an OQS model outputs probabilities for different options. Which option is selected for the response is stochastically determined and even low probability options can be produced.

Third, drift rates for different options can be separated out into several components, which are independently manipulated. In the specific OQS model we propose, there are independent evidence accumulation processes from uncertainty to each of the two options and conversely from each option to uncertainty – there are no direct accumulators connecting each option (no inhibition). We think this is intuitive. I may think option A is good and option B is bad. But it does not have to be the case that the badness of B necessarily translates to goodness for A. We can also more flexibly manipulate the number of distinct states. In a binary decision task, there are two obvious states for preferring one option vs. the other, and this would be the standard approach in a drift diffusion model too. However, a restriction to just these two states means that, if a person is asked to make a decision, the person will always have to respond with a yes or a no. We think that for at least some real-life decisions this would be implausible. Instead, there may be cases when, if a person is asked to make a decision, she will conclude that she is uncertain, rather than commit to a yes or a no, that is, decide to be undecided or defer a decision (e.g., Dhar and Simonson, 2003). Note, this particular feature of the model (having three states, one corresponding to uncertainty) is also required for technical reasons. Specifically, a simpler, two-dimensional OQS model of this kind produces dynamics which always stabilize at $\rho_{11} = \rho_{33} = 0.5$.

Fourth, evidence accumulation towards particular options is balanced against the degree of intrinsic oscillation in the system – this balance corresponds to the conflict between the Schrödinger part of the Lindblad equation vs. the Lindblad part. When the influence of the Schrödinger part is too strong, stabilization towards an end state might be delayed or never happen within the available time. A main distinctive feature of an OQS model is exactly this capacity to produce oscillations, that is, periodic changes in attentional (or decisional) focus concerning which option is favored or how strongly an option is favored.

Fifth, an OQS model can allow drift towards several distinct states, whereas standard drift diffusion models typically allow drift only towards two options (Ratcliff & Smith, 2015). This would be well suited to situations where there are multiple utilities or several options.

Overall, there is no doubt that diffusion models have contributed enormously to our understanding of decision dynamics. Here, we hope to have provided sufficient motivation to explore an architecture for dynamics with different form and various unique features.

1.2. An OQS model for attentional dynamics in bivalent preference

Consider a choice between two unmatched alternatives, for example, whether to keep a stray dog you found or not. There are pros and cons for keeping the stray dog, but no pro can be directly linked to a con. In our experimental paradigm, the pros and cons were visually presented to participants concurrently, in distinct Areas of Interest (AOIs). Our aim is to develop a model for the dynamics of attention towards the two AOIs. We expect attentional dynamics to reflect some (possibly limited) oscillation, as attention shifts to reflect consideration from cons to pros and vice versa, possibly followed by stabilization, when the gaze cascade effect takes over (Shimojo et al., 2003). In this section we outline the models which we developed for this problem. We first offer a conceptual outline of the two models, followed by a (fairly) condensed presentation of the technical details.

We define a three-dimensional space, such that each of the three dimensions corresponds to one of three possible outcomes of the decision task: favor the first option (accept the proposal), favor the second option (reject the proposal), or uncertainty (declare you cannot decide). As noted, this picture allows the model to predict that the outcome of the decision process might be uncertainty about which option should be preferred (the decision maker would be certain she is uncertain between the two options, cf. Dhar & Simonson, 2003). The three dimensions in the space are called a basis set. In quantum theory, a basis set always represents the outcomes of a particular question, in the present case 'which option do you prefer?'. Clearly, there are different questions one can ask and so different basis sets. In the present case, we only consider one basis set, because we are interested in only one question.

A mental state in this space can correspond to a vector, with amplitudes along each of the three dimensions or, more generally, a density matrix, which can be seen as a linear mixture of state vectors. Dynamics concern the way the state vector or density matrix can change with time. Schrödinger's equation is expressed more familiarly with state vectors, but Lindblad equation with density matrices. The crucial point is this: when a density matrix is diagonal, then the corresponding state can be considered classical, relative to the basis set we are employing (the subtlety here is that relative to another basis set, the same density matrix may not be classical; this is not presently relevant since, as noted, we are only interested in a particular question and so only one basis set). Off-diagonal terms in the density matrix indicate quantum character (again, this statement is relative to a particular basis set; henceforth, we will stop making this qualification). Therefore, the extent to which there is oscillation/ ambivalence vs. stabilization in the dynamics depends on whether there are only diagonal vs. off-diagonal terms in the density matrix.

Schrödinger's equation can create off diagonal terms in the density matrix even if the density matrix is initially diagonal. That is, even with an apparently classical state to start with, quantum character and oscillations can emerge early on. The main element in Schrödinger's equation is the Hamiltonian, which is a transition matrix for how amplitude towards each of the three outcomes gets shuffled around with time (the Hamiltonian is analogous to the intensity matrix in the Kolmogorov equation). Lindblad's equation for OQS has a part which is identical to Schrödinger's equation and an additional part, the Lindblad term. The Lindblad part can gradually eliminate off-diagonal terms and so quench any oscillations. As noted, this is the process of stabilization or decoherence and its strength (and whether it will be possible at all) depends on the relative strength (parametrically determined) of the Schrödinger part vs. the Lindblad part. The Lindblad part is built using the so-called C operators. Which C operators are included in a particular situation is a modeler's choice and each C operator is weighted by a coefficient. In the simplest case, C operators can be specified so that, each one of them, can be thought of as driving stabilization towards a particular question outcome. Therefore, the coefficients of C operators are analogous to drift rates in diffusion models, with the qualification that a model can involve several C operators (and so several drift rates).

A quantum dynamical model, whether based on Schrödinger's equation or the Lindblad equation, can be used at each point to compute the weight towards each of the options (the probability of each option). For the present purposes, we can use such models to predict the relative weight of attentional focus towards the first or the second option, in each of the decision scenarios participants were faced with.

The mathematical specification of the OQS models we employed follows the intuitive presentation above and we summarize it here (additional details in Appendix A). Even though some of the mathematics may be unfamiliar in our field, essentially most of the computations are straightforward linear algebra.

The canonical (preferred) basis for the situation of interest involves three basis vectors, defined as $|yes\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $|undecided\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $|no\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$, where $|.\rangle$ indicates a column vector, and the yes, no outcomes concern the choice offered to participants (accept vs. reject a proposal). If the mental state is written as $|\psi\rangle = a|yes\rangle + b|undecided\rangle + c|no\rangle$, the coefficients a, b, and c are the weights for each of the three decision outcomes, called amplitudes. Such a state is called a superposition and it embodies quantum character, for example, a decision can change the state (cf. White et al., 2020). The mental state can be also expressed as a density matrix, ρ, which is a positive, semi-definite operator ($\langle\psi|\rho|\psi\rangle \geq 0$, for all vectors $|\psi\rangle$), with $\rho = \rho^\dagger$ and $Trace(\rho) = 1$, where $\dagger$ indicates the conjugate transpose of a matrix and the trace operation sums the diagonal elements of a matrix (the latter condition is a probability normalization one). In a three-dimensional space, ρ would be a three-dimensional matrix. For the present case, $\rho_{11} = 1$ corresponds to 'yes', $\rho_{33} = 1$ to 'no', and $\rho_{22} = 1$ to 'undecided'. Whether ρ is diagonal vs. whether it has non-zero off-diagonal terms can be interpreted in terms of ρ being a classical state vs. including quantum character. Specifically, ρ can be expressed as a linear mixture such as $\rho = \sum_i c_i P_{\psi_i}$, where $P_{\psi_i} = |\psi_i\rangle\langle\psi_i|$, $\langle\psi_i|$ is the conjugate transpose of $|\psi_i\rangle$, and the c_i's are classical probabilities. For example, if $|\psi\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, then $|\psi\rangle\langle\psi| = \begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}1 & 0 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{pmatrix}$. So when ρ is diagonal it is a linear mixture of basis vectors and when ρ has non-zero off-diagonal terms a linear mixture of general superpositions. The gradual lossof quantum character is the hallmark of OQS and goes hand inhand with a transition from oscillatory dynamical behavior to stabilization.

For isolated systems, dynamics is governed by Schrödinger's equation, $\frac{d|\psi(t)\rangle}{dt} = -iH|\psi(t)\rangle$, which solves to $|\psi(t)\rangle = e^{-i \cdot t \cdot H}|\psi(0)\rangle = U(t)|\psi(0)\rangle$. Matrix H, the Hamiltonian ($H^\dagger = H$), is a transition matrix which determines which elements of $|\psi(t)\rangle$ increase or decrease in amplitude. $U(t)$ is a unitary operator, i.e., $U^{-1} = U^\dagger$. With density operators, Schrödinger's equation is $\frac{d\rho}{dt} = \dot{\rho} = -i[H, \rho]$, where the bracket notation indicates the commutator, $[A, B] = AB - BA$. The specification of the Hamiltonian H is the key aspect of modeler insight concerning the empirical situation at hand. For example, for a prisoner's dilemma game, Pothos and Busemeyer (2009) specified a Hamiltonian so that its parameters corresponded to utilities for the various options and a mechanism of cognitive dissonance. Presently, we propose a Hamiltonian with parameters whose relative weight governs transfer of amplitude from either of the certain options to the uncertain option and back – this is how we aimed to capture psychological ambivalence. Specifically, we propose $H = \begin{pmatrix} \mathrm{E0} & d & 0 \\ d & 0 & d \\ 0 & d & \mathrm{E0} \end{pmatrix}$. The three eigenvalues of H are $\{\mathrm{E0}, \frac{1}{2}(\mathrm{E0} - \sqrt{8d^2 + \mathrm{E0}^2}), \frac{1}{2}(\mathrm{E0} + \sqrt{8d^2 + \mathrm{E0}^2})\}$, so the dynamical process will take place at two speeds, a slow one of order (for frequency) $\frac{1}{2}(\sqrt{8d^2 + \mathrm{E0}^2} - E0)$ and a fast one of order $\sqrt{8d^2 + \mathrm{E0}^2}$. For simplicity, we can consider E0 a time-scale constant and set it to 1.

The Lindblad equation is $\dot{\rho} = -i[H,\rho] - \mathscr{L}(\rho)$, with $\mathscr{L}(\rho)$ being the Lindblad term. When $\mathscr{L}(I) = 0$, then the Lindblad operator is called unital and it can be shown that purity (i.e., degree of quantumness) is monotonically decreasing with time (Lidar, Shabani, & Alicki, 2006). We can write $\mathscr{L}(\rho) = \sum_j \Gamma_j \left(C_j \rho C_j^{\dagger} - \frac{1}{2}\rho C_j^{\dagger} C_j - \frac{1}{2} C_j^{\dagger} C_j \rho \right)$, where the Γ_j scalar parameters determine the strength of interaction with the environment (these parameters are absorbed into the 'a' parameters for the Lindblad term, see just below). The C_j operators are three-by-three operators (matrices) that determine the specific environmental interactions. As noted, particular C_j operators drive stabilization towards particular states. In the present situation, we suggest that there are at most four psychological processes, drifts from uncertainty to certainly for either of the two options (yes or no) and drift from certainty (either for yes or no) to uncertainty. The first two processes straightforwardly correspond to the decision maker trying to make up her mind. But we might also observe decision makers whose early certainty towards the yes or the no options is undermined, perhaps as they consider different arguments for the options, motivating the inclusion of drifts from certainty to uncertainty.

Mathematically, drift from uncertainty to the yes and no options correspond respectively to $C_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $C_{32} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$; drift from certainty for yes to uncertainty and from certainty for no to uncertainty correspond to, respectively, $C_{21} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $C_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; note, the indices here are no longer summation indices, but reflect the states across which there is drift. To simplify notation for these C operators in the Lindblad equation, define $\Lambda(\rho, C_{xy}) = \left(C_{xy} \rho C_{xy}^{\dagger} - \frac{1}{2}\rho C_{xy}^{\dagger} C_{xy} - \frac{1}{2} C_{xy}^{\dagger} C_{xy} \rho \right)$, noting that this just follows from the $\mathscr{L}(\rho)$ expression above. Each C operator is weighted by a parameter which informs the strength of the corresponding drift process. Depending on whether the drift processes only concern transfer from uncertainty to certainty or certainty to uncertainty as well, the Lindblad part is $a_{12}\Lambda(\rho, C_{12}) + a_{32}\Lambda(\rho, C_{32})$ or $a_{12}\Lambda(\rho, C_{12}) + a_{32}\Lambda(\rho, C_{32}) + a_{21}\Lambda(\rho, C_{21}) + a_{23}\Lambda(\rho, C_{23})$. Parameters a_{12}, a_{32}, a_{21}, a_{23} are analogous to drift rates in a standard drift diffusion formalism

Overall, the two versions of the OQS models we developed, referred to as OQS4 and OQS6, are:

$$\dot{\rho} = -i[H,\rho] + a_{12}\Lambda(\rho, C_{12}) + a_{32}\Lambda(\rho, C_{32}) \tag{1}$$

$$\dot{\rho} = -i[H,\rho] + a_{12}\Lambda(\rho, C_{12}) + a_{32}\Lambda(\rho, C_{32}) + a_{21}\Lambda(\rho, C_{21}) + a_{23}\Lambda(\rho, C_{23}) \tag{2}$$

The Hamiltonian, H, and the C operators are as specified above. Overall, OQS4 has four parameters, one for the Hamiltonian (d), two for the drift processes (a_{12}, a_{32}), and a parameter to stretch or compress the time scale. OQS6 has six parameters, all the parameters of the simpler model and the ones for the additional drift processes (a_{21}, a_{23}). In both cases, the prediction from the quantum models for attentional focus (across time bins) for the yes/ pros AOI is ρ_{11} (i.e., the '11′ element of the density matrix) and for the no/ cons AOI ρ_{33}. The optimization procedure (using the NonlinearModelFit function in Mathematica) requested soft bounds for the drift parameters in the [0, 3] range, d in the [0.1, 0.9] range, and the time scale parameter in the [0.1, 1.5] range.

An important detail of the model concerns the initial state, because this initial state interacts in a complex way with both the Schrödinger and the Lindblad parts of the dynamics. The experimental procedure we adopted involved two decisions. For the first decision, it is natural to require an initial state of uncertainty, $\rho = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. For the second decision, on the one hand, it might seem reasonable to set the state to the outcome of the first decision. In quantum cognitive models, typically once a decision is made, the state has to align with the outcome of the process (White et al., 2014, 2020; Yearsley & Pothos, 2016). However, such quantum-like constructive influences would only apply when a sequence of decisions are back-to-back, since any intermediate cognitive processes would change the state (see, e.g., Wang, Solloway, Shiffrin, & Busemeyer, 2014). Additionally, in the present paradigm participants were asked to reconsider the problem for the decision, to rethink it from scratch. Therefore, for the second decision too, we opted for using as the initial state the uncertainty state above. Note, we fitted the OQS4 and OQS6 models both with initial states as described here and with initial states such that for the second decision the state was set to the outcome of the first one. With the latter case, fits for the OQS6 model were worse than for the null model, so we retained only the fits for the former case.

The OQS framework is probably unfamiliar to psychologists (Asano et al., 2011; Kvam et al., 2021). It may look like the implementation of the OQS4, OQS6 models is post hoc. There are certainly many exploratory aspects to the present work, notably the

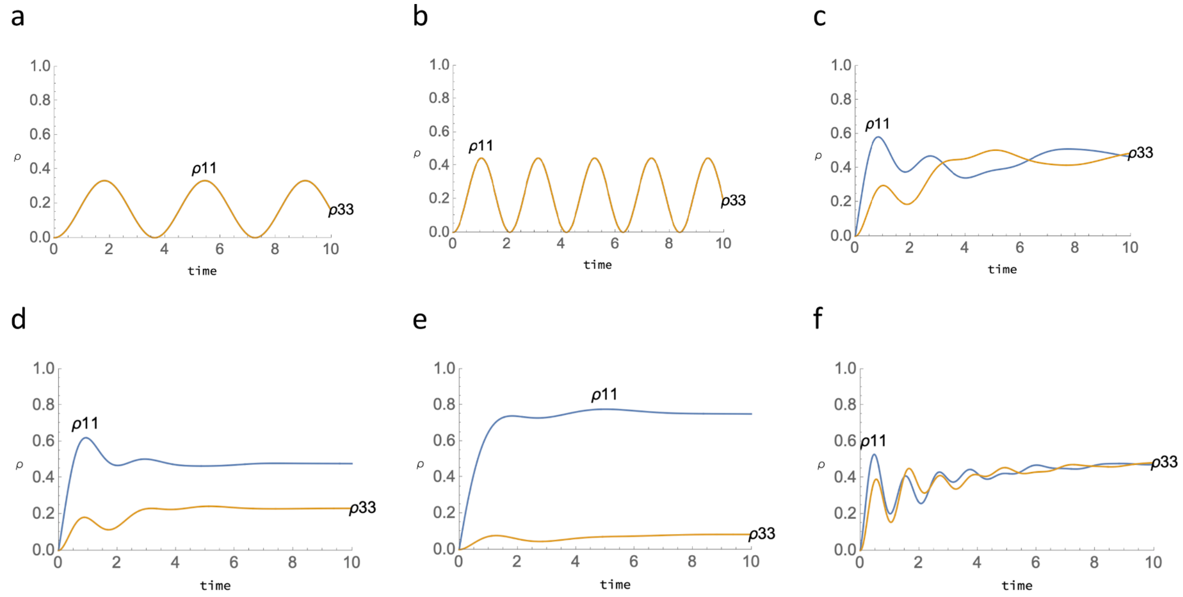

Fig. 1. Examples of OQS6 model behavior. In all graphs, the orange curve shows ρ_{33} and the blue one ρ_{11} (where there is only an orange curve, this is because of perfect overlap). The vertical axis is probability and the horizontal axis time; reaction time was set to 10 (arbitrary scale). For successive panels, we only note the change relative to the previous panel. Panel (a): d = 0.5, all drifts = 0. Panel (b): d = 1. Panel (c): $a_{12} = 1$. Panel (d): $a_{23} = 1$. Panel (e): d = 0.5 (relative to panel (d), observe how reducing the strength of oscillations results in stabilization at more extreme levels). Panel (f): $a_{12} = 1$, $a_{23} = 0$, d = 2 (this illustrates gradually quenched oscillations).

proposal that an OQS model is relevant in the specific behavioral situation. Also exploratory is whether attentional dynamics can tell us anything interesting about an ambivalent or otherwise decision problem, as well as the best way to measure attentional dynamics. It is fair to say that there is not much prior work on ambivalence and so available relevant guidance. With these thoughts in mind, the specification of the OQS models was done by adopting assumptions which were as minimal as possible.

The behavioral problem concerns drift in attentional focus with time, across two competing options, Yes vs. No. We allowed for three possibilities: (a) attention towards Yes dominates (b) attention towards No dominates (c) attention towards neither response dominates (so that the decision process concludes with uncertainty). Because we allowed for these three possibilities, the minimum required dimensionality for the quantum space is three, and this is the approach we adopted. The initial state for both decisions was set to coincide with the uncertain state. A more elaborate option would be to parameterize the initial state. The Hamiltonian in the OQS model is specified with one parameter concerning the oscillation between the Yes, Uncertain states and the same parameter between the No, Uncertain states. There are several extensions to this basic approach, as the only requirement for the Hamiltonian is that it is Hermitian (it is equal to the complex conjugate of its transpose). The Lindblad part involves two drifts in OQS4 (from the uncertain state to Yes and from the uncertain state to No) and two more drifts in OQS6 (from Yes to the uncertain state and from No to the uncertain state). Overall, we opted for the simplest approach matching the behavioral situation.

Generally, the model behavior embodies a conflict between the intrinsic oscillation mechanism from the Schrödinger part and drift processes from the Lindblad part. Psychologically this is the novel proposal, that together with drifts towards particular options (presumably determined by the decision maker's perception of the strength of the different arguments), there is also an intrinsic process of uncertainty, which can undermine or entirely prevent these drifts. Other model characteristics closely align with those in standard drift diffusion models. Notably, the model has no component for memory storage or retrieval. Instead, all the information accumulated about the various options is encoded in the state, ρ. So, at any given time, the way the state is set up reflects the bias for the decision maker to focus on the different options, based on the information she has processed so far. The model includes a component for attentional allocation, since at any given time point, we can examine the state ρ and infer the probability that attention will be allocated to the different AOIs. Also, the drift parameters can be interpreted in terms of an information integration mechanism, which determines how the evidence for each option is weighted.

The final consideration concerns expectations for model behavior. For different model configurations, oscillation may not occur at all or may be prolonged indefinitely; stabilization may occur at different levels balancing the competing influences in the model. For example, $a_{12} > a_{32}$ would be interpreted as higher evidence for the first option, so that over time we might expect oscillations to be quenched so that $\rho_{11} > \rho_{33}$. However, the extent to which the expected stabilization behavior occurs or not will also depend on the intrinsic oscillation in the system. Higher values of d (faster oscillation) mean that stabilization may end up at the uncertain state, even if $a_{12} > a_{32}$. Psychologically, we interpret this as a conflict between evidence for one option against the other vs. intrinsic ambivalence regarding the decision; where the latter is high, the cognitive agent will be unable to develop sufficient preference for one option vs. the other, even at long times. Also, stabilization towards one option is aided both by drift from uncertainty to that option and drift from the other option to uncertainty, in the OQS6. We interpret this as very strong preference for an option separately requiring both support for

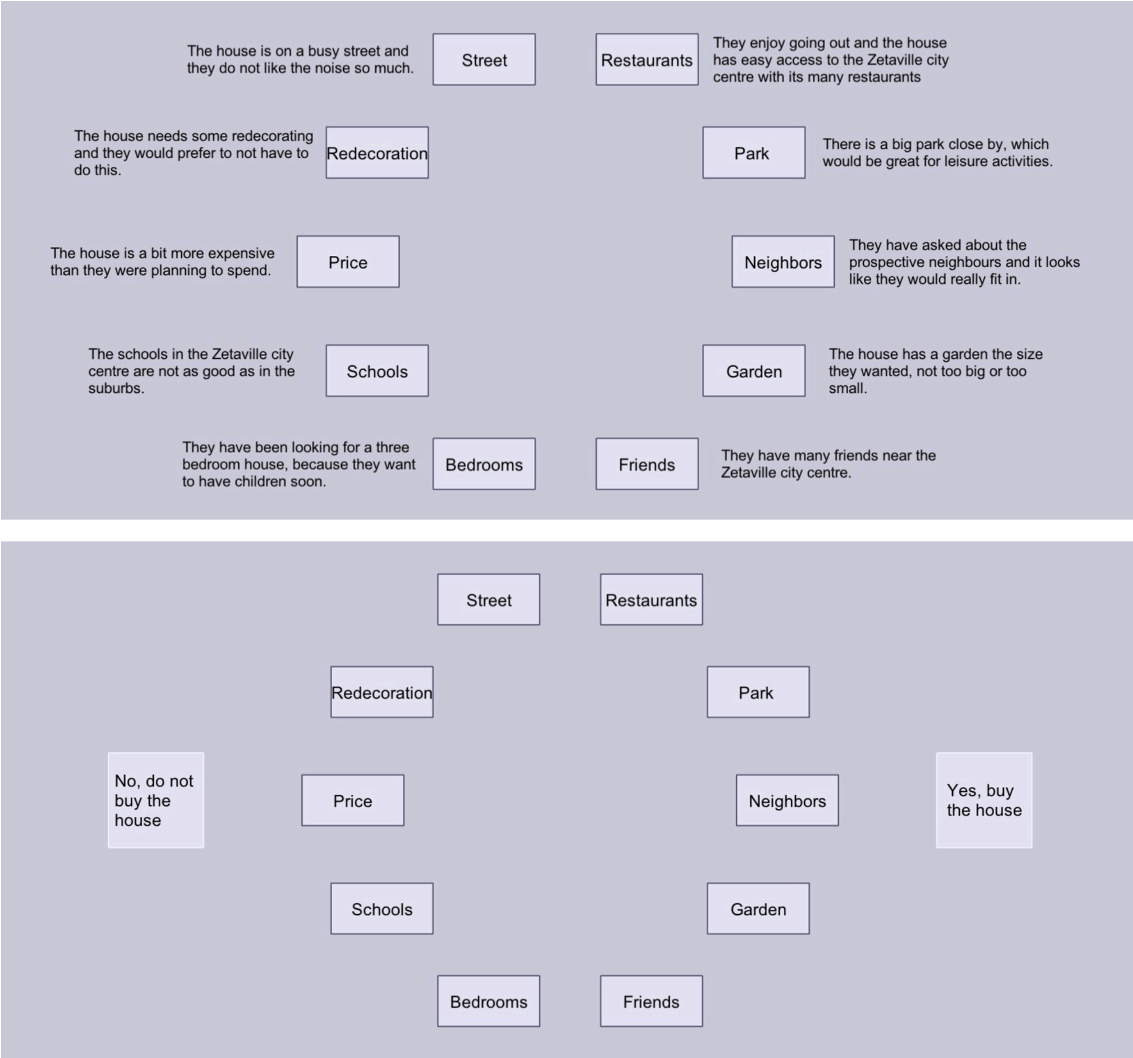

Fig. 2. Spatial layouts in the pre-decision stage (on the top) and the decision stage (on the bottom), for the first scenario, buying a house. Corresponding figures for the other two scenarios (going to the cinema; keeping a stray dog) and the full text of the statements are shown in Appendix C.

the option and lack of support for the other option.

In Fig. 1 we briefly illustrate these ideas, for OQS6. We start with an uncertain initial state, $\rho_{22} = 1$. In the top left panel, we consider d = 0.5, with all the a parameters set to 0 – there is a straightforward picture of indefinite oscillation. In the top middle panel, increasing d increases the frequency of oscillations. In the top right panel, setting $a_{12} = 1$ produces some stabilization, but it is not sufficiently strong to discriminate between the yes, no responses. To get stabilization to strongly discriminate between yes, no, one needs both $a_{12} = 1$ and $a_{23} = 1$ (middle bottom panel).

1.3. Outline of the empirical paradigm

We sought an instance of a vague, complicated choice, involving several poorly matched attributes, that are not easily quantifiable, such as choosing between a jumper and a shirt at a store or trying to decide whether to buy a new car or make an overpayment on a mortgage. In such cases, there is an intuitive expectation of ambivalence, such that at different times we may appear close to deciding in favor of one option and at other times the other option (for related work on ambivalence in attitudes see also Nohlen, van Harreveld, Rotteveel, Lelieveld, & Crone, 2014; van Harreveld, Nohlen, & Schneider, 2015; Van Harreveld, Van der Pligt, & de Liver, 2009). This paradigm was selected to increase external validity, but also the likelihood that oscillations can be observed. In previous research using standard decision-making paradigms, in which attributes can be directly compared, usually only a few fixation transitions between options are observed. In a standard risky choice paradigm, for example, with choices between two gambles with two outcomes each, only three to four transitions between the gambles per choice were observed (Glöckner & Herbold, 2011), which might suggest a low number of oscillations too. However, note that oscillations refer to changes in focus towards one option vs. the other, which could also occur within AOIs (cf. Fig. 1f; we later provide a more precise way to operationalize our notion of gradually quenched oscillations).

We created three scenarios, referred to as 'buying a house' or just 'house', 'going to the cinema' or 'cinema', and 'keeping a stray dog' or 'dog'. In each case, participants were asked to consider a proposal from the perspective of two hypothetical individuals, based on a list of pros vs. cons. There was the same number of pros and cons and the weight of pros was approximately equivalent to cons, but otherwise they were not matched (some piloting was carried out to achieve better balance between the pros, cons, see Appendix B). Participants had to either accept the proposal, responding with a 'yes', or reject it, responding with a 'no'. We expected participants to spend some time vacillating, before a final decision.

Participants were initially allowed some time to process the pros and cons for a scenario. They were then presented with a spatial layout for the pros/ cons and a label associated with each argument, so that the pros and cons could neatly be divided into separate AOIs. Subsequently, the full statements for each pro and con were removed, while the corresponding labels remained visible, and in addition response buttons were introduced that participants would have to click on eventually, to indicate their decision (Fig. 2). It is at this point that we started recording eye fixation information. The main dependent variable is fixation dynamics until a decision is made, computed as proportions of fixations to the pros (yes) AOI, cons (no) AOI or neither (undecided), within time bins of a certain size.

Following the first decision, participants were told they had to reconsider the decision problem and provide a second answer; the first decision was self-paced, but for the second decision participants were given a fixed amount of time (15 s) to deliberate. There are a few reasons why we wanted to include a second decision. First, we were interested in both allowing participants to a decide in their own time (which would be the more natural way to reach a decision) and to deliberate over a fixed period of time (which might offer more opportunity for back-and-forth). Second, as this was an experiment on ambivalence, we thought it would be worthwhile to examine dynamics, having asked participants to reconsider their decision. In this second decision, there may be a degree of ambivalence from having to reconsider the initial decision. Finally, we wanted to collect more data from participants.

The empirical approach assumes that it is possible to differentiate between a strictly information processing step (reading through the pros and cons) and the decision step (balancing out the pros and cons to reach a decision). However, as soon as participants start processing the information, it is likely that they will also evaluate it and start being biased for particular decisions (regardless of any instructions to not do so). It seems impossible to separate a process of reading the arguments from the decision process, and by the time participants reach the eye tracking stage they may have already made some progress towards deciding. Another disadvantage is that the actual labels themselves may generate attentional biases (Cox et al., 2006). For example, a participant who may have recently lost a beloved pet would plausibly display disproportionate high fixation counts for particular pros, cons in the corresponding scenario. Memory retrieval processes might also be confounding the decision processes, that is, fixations may reflect attempts to retrieve information from memory, rather than a process of trying to balance cons vs. pros. However, self-paced presentation of the pros and cons in the pre-decision step should make it more likely that there is good memory encoding. Overall, the disadvantages of distinct encoding vs. decision steps have to be judged against the advantage of having a paradigm which allows process tracing of the attentional dynamics. Additionally, the paradigm should reduce influences on attention due to information search (e.g., just reading and trying to understand the arguments), and thus attention may be a more direct measure of the preferential state. A pretest for the pros and cons in each scenario is described in Appendix B.

2. Experimental investigation

2.1. Participants

Forty-five students of University of Zurich (10 male, 35 female; age $M = 21.58$, $SD = 4.28$, $range = 18$–42) participated in the study. Participants received course credit or monetary compensation (7.5 CHF) for participating in the study. All had normal or corrected to normal vision. All were native German or Swiss German speakers. We did not use pre-registration in this study since we followed a mainly exploratory approach that aims at model development and fitting. For the same reason, we did not conduct an a priori power analysis to determine sample size. A post hoc power analysis revealed that with our sample medium effects could be detected with 1 – beta = 0.96 ($f = 0.25$, repeated measurement ANOVA, within-participants factor, Faul et al., 2007).

2.2. Apparatus

Participants were seated in front of a 22-in. computer screen (1,680 × 1,050 pixels) at a distance of 700 mm and instructed to position their head in a chin rest. Stimuli were presented using PsychoPy 1.84.1 running on a separate computer. The eye tracker system SMI iView RED sampled data of the right eye at 500 Hz and recorded with iView X 2.8 following a 5-point calibration. Auditory signals were presented over loudspeaker. Participants responded with mouse clicks. Fixation detection was performed with the IDF Event detector 3.0.20 (SMI, Teltow) using a peak velocity threshold of 60°/s and a minimum fixation duration of 40 ms.

2.3. Materials

Participants were presented with three decision scenarios. In all cases, a young couple was faced with a binary decision, to buy a house, to adopt a stray dog, to go to a movie. Participants were called to make a 'yes,' 'no' decision regarding a question in each scenario, based on a list of pros and cons. All materials were presented in German – English versions of the scenarios and the pros, cons

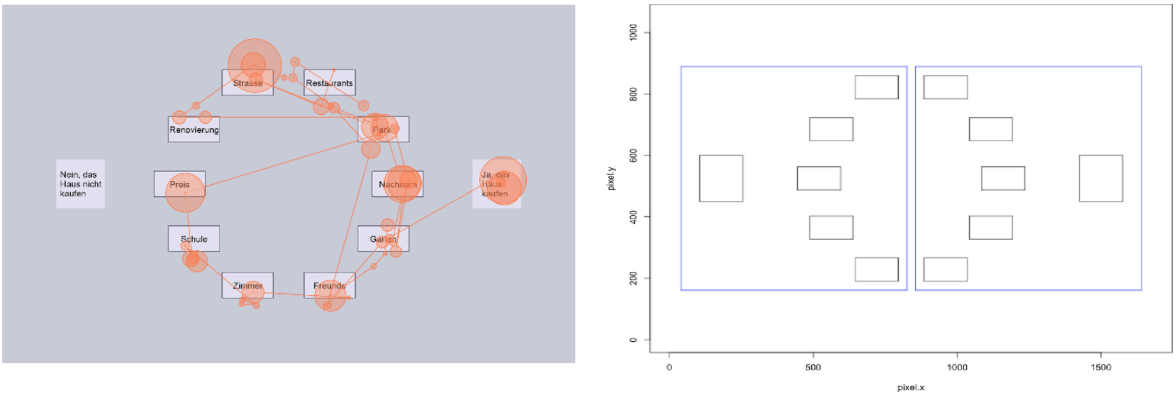

Fig. 3. A sample scan path on the left and the AOIs employed in the eye-tracking analysis on the right.

are shown in Fig. 2 and in Appendix C. Appendix C also contains the originals in German (the two versions had slight differences).

Participants worked through the decision for each scenario one at a time. Participants first read the scenario. This was followed by a screen with all the pros and cons, so that each pro was presented together with a label summarizing the argument and contained within a rectangle. These labels were arranged on a circle, so that all labels had the same distance from the center of the screen (Fig. 2, left). Each rectangle had a size of 2.55° × 1.28° of visual angle (150 × 75 pixel) and the circle had a distance of 4.61° of visual angle (320 pixel) from the center of the screen. The pros and cons themselves were each presented to the left or right side of each rectangle with a distance of 8.50° of visual angle (590 pixel) from the center of the screen. The intention of this screen was to enable participants to read through the pros and cons and associate the pros, cons with their labels.

The next screen eliminated the full statements for each pro or con and retained only the labels (Fig. 2, right). Additionally, there were response buttons on the left and right side of the screen, which started with yes, no prompts, and were followed by some text reminding participants of what each response corresponded to (e.g., “Yes, go to the cinema, tonight” and “No, don’t go to the cinema, tonight.”). The buttons had a size of 2.02° × 2.02° of visual angle (140 × 140 pixel) and were located 8.78° of visual angle (610 pixel) from the center of the screen (see Fig. 2, right). The background of the screen was gray to prevent eye strain. The spatial location at which a particular pro or con appeared was randomized across participants; whether the group of pros (or cons) was presented on the left or the right was also randomized across participants, to control for biases in initial fixations. Finally, the order of the three scenarios (House, Dog, Movie) was randomized.

2.4. Procedure

After participants were welcomed by the experimenter and signed the ethics consent form, they went through a short eye-tracker test, which involved a practice scenario (designed in a way analogous to that for the main ones). This practice scenario was included to familiarize participants with the procedure and to allow calibration of the eye-tracker (this was done following this practice scenario).

The procedure was identical for each of the three scenarios. First, participants clicked the mouse to view the story corresponding to the current scenario, without the pros and cons, in a self-paced manner. Participants indicated they had sufficiently read the story by clicking the mouse, at which point a fixation cross appeared for 1.5 s. Second, participants saw the pros and cons, as in Fig. 2, left, also in a self-paced manner. Participants were told to read through the pros and cons and when ready to click the mouse, to proceed to the decision step. Third, participants saw the pros and cons labels (without the full statements for each pro or con), as in Fig. 2, right, together with the prompts to respond yes or no (depending on the scenario, there would be a slight elaboration of what yes or no meant). This decision step was self-paced and was terminated when participants clicked on either the yes or no button. There is a question here concerning whether the labels offered adequate reminders for the arguments. In a post-experiment questionnaire, we presented all cues and arguments to participants. Their task was to indicate how important they found the argument for their decision or to indicate if they have forgotten the argument. Only in 5 out 1350 cases (45 subjects × 3 stories × 10 arguments) participants indicated that they had forgotten what the argument meant. Note, presentation in this post-experiment questionnaire involved both the argument and the cue. So, it is possible that the rate 5/1350 underestimates the true lack of retrieval rate, during the experiment (when only the cues were shown). Still, we think these results are encouraging regarding the ability of our participants to remember the arguments during the experiment.

Fourth, once participants had made their first decision, there was a second decision step, which involved the same screen as before (Fig. 2, right) and required participants to view the pros and cons for 15 s, before making a second decision. Participants were alerted when 15 s had elapsed by a tone, though note that following the tone participants might still take a few seconds to respond. Once this second decision was made, to prevent an abrupt offset of eye movement recordings, the decision screen was visible for another 500 ms. Fifth, participants were asked to indicate their confidence for the decision on a Likert scale, anchored at 1 (very unsure) and 9 (very sure). Sixth, participants were presented with four questions (anchored at 1 and 10), which queried participants regarding their reliance on personal experience to make their decision (as opposed to trying to think what the protagonists in each scenario should do),

Table 1
Summary descriptives for key variables in the study.

Variable	Decision 1	Decision 2
Decision response (% "Yes")	42.2	41.5
Decision time(sec)	14.9 (14.5)	17.5 (2.48)
Transitions	13.43 (11.79)	16.15 (6.51)
Transitions within	10.47 (8.93)	12.64 (5.44)
Transitions between	2.96 (3.67)	3.51 (2.59)
Strategy Index (SI)	0.61 (0.32)	0.56 (0.24)
AOI_{pros}	0.42 (0.25)	0.45 (0.19)
AOI_{pros3}	0.43 (0.36)	0.44 (0.38)
$AOI_{pros(start\text{-}3)}$	0.43 (0.27)	0.46 (0.19)

Note. Values are given as *M* (*SD*). Transitions within refers to transitions within each AOI and between across AOIs.

the personal relevance of the decision, their experience with such decisions, and the effort they made to reach a good decision (Appendix D). We refer to these four variables as motivation variables.

Once participants had viewed all three scenarios, they were asked to rate the importance of each pro and con for each story, on a Likert scale anchored at 1 (not important at all) and 10 (extremely important). For each pro or con, they could also indicate whether they had forgotten an argument. The study lasted on average $M = 15.84$ min, $SD = 2.68$; mean tracking accuracy was $M = 0.61$ degrees of visual angle, $SD = 0.17$ (there was a threshold of 1 degree of visual angle to pass the calibration routine).

2.5. Eye movement analysis

Unsurprisingly, there was a complex interplay between fixations towards particular pros, cons and the yes, no buttons (e.g., Fig. 3, left). To make the analytical task tractable, we considered vacillation in attentional dynamics in broad terms, by specifying two large rectangular AOIs, to include the pro arguments and the yes button on the one hand, and the con arguments and the no button on the other (Fig. 3, right). Each AOI had a size of $11.30^\circ \times 10.48^\circ$ of visual angle (785×728 pixel). The size of the AOIs exceeded the top and left (right AOI) versus right (left AOI) borders of the top and bottom argument rectangles by 0.43° of visual angle (30 pixel), leaving a gap between the large AOIs of 0.43° of visual angle (30 pixel).

We analyzed the sum of fixation durations to the two AOIs within 100 ms time intervals from the beginning of the trial until the response. If the sum of fixation durations for one interval did not sum up to 100 ms, this was due to fixations outside the AOIs, saccades or blinks (e.g., eyes closed). For the model fitting, after smoothing (see below) we averaged successive bins to produce an effective bin of 200 ms. The bin size of 200 ms was chosen as a compromise between assumptions regarding the lower limit of the granularity of the underlying process (fixations can be as short as 40 ms) and expectations about noise which would recommend measurement within larger bins.

3. Standard statistical analyses

On average participants did not have a clear preference for a 'yes' or 'no' response and took about 15 sec to make a decision for the first decision (for which there were no time constraints), suggesting that we reached the goal of a fairly complicated and ambivalent choice (summary statistics are reported in Table 1). In both cases, transitions correlated with response time (for the first decision, $r = 0.83$, for the second $r = 0.29$; $N = 135$ and $p = .001$ or lower). We computed the Strategy Index (SI; Payne, 1976), which is the difference between within AOI and between AOI transitions, divided by all transitions, as a measure of the preponderance within (SI = 1) vs. between (SI = -1) option processing. For the first and second decision respectively, results indicated more within option that between option transitions. It is possible that the number of transitions might relate to one of the four motivation variables (Appendix D). This was indeed the case for transitions in the first decision and the variable concerning the amount of effort that went into the decision ($r = 0.2$, $p = .02$, $N = 135$), but no other associations were identified regarding transitions in the first or second decision and the motivation variables.

Overall, we did not observe as many between-AOI transitions as we were expecting (Table 1), even though we were intending to create an experimental paradigm which would lead to many transitions (at any rate, oscillations) in attentional dynamics. Retrospectively, we can recognize some reasons for this, including the particular format in which the information was presented to participants. With a circular arrangement, the distance between the two options was larger, than if the information were presented in a table-like format, which possibly reduced the number of transitions between options. For instance, Fiedler and Glöckner (2012) used a similar visual layout and found comparable numbers of between option transitions (see also Glöckner & Herbold, 2011).

As expected, the two decisions correlated, $r = 0.86$, $p < .0005$, $N = 135$, so here we focus on the first decision (modeling will involve both decisions). First, we consider whether the behavioral variables can predict the decisions. Given that we have multiple responses per participant, we ran a mixed effects binary logistic regression analysis, using the Generalized Linear Mixed Model procedure in SPSS, with first decision as the dependent variable, participants as a random effect (modeled only with intercepts, no slopes; best model identified using BIC), and scenario and signed average pros, cons difference as fixed effects (the latter variable is the average rated importance of pros for a scenario minus the average rated importance of cons for a scenario). Both fixed effects were significant, scenario ($F(2,129) = 4.40$, $p = .014$) and the signed average pros, cons difference ($F(1,129) = 37.39$, $p < .0005$), but not the interaction ($F(2,129) = 1.20$, $p = .31$). The uncorrected correlation between the first decision and signed average pros, cons difference was −0.74 (note, 'yes' was coded with a 1 and 'no' with a 2). These results provide some confirmation that decisions were largely driven by participants' impressions of the strength of different pros, cons in each scenario (a qualification to this conclusion is that participants might be adjusting their ratings post-decision to be more in line with the decision). Hence, in line with previous work (e.g., Bröder, 2003; Brusovansky, Glickman, & Usher, 2018; Glöckner & Betsch, 2008; Glöckner, Hilbig, & Jekel, 2014, participant choices could on average be well described by a Weighted Additive Strategy according to which people choose the option with the higher importance-weighted attributes (Payne, Bettman, & Johnson, 1988).

We next considered whether attentional dynamics can be related to behavioral variables, specifically whether perceived strength of pros vs. cons might be driving attention; and evidence for a link between eye movements and decisions (a gaze cascade effect). AOI_{pros} refers to the sum of fixation durations for the pros AOI, normalized by all fixation durations to either the pros or the cons AOI, for the first decision (note, this is not just the overall reaction time, since there will be fixations outside the two AOIs). A corresponding variable for the second decision correlated reasonably highly with the first ($r = 0.43$, $p < .0005$, $N = 135$), so we continue focusing on the first decision. We also computed a similar variable, AOI_{pros3}, for the last three seconds leading up to a decision; and $AOI_{pros(start\text{-}3)}$, covering the time period from start to three seconds before the decision (this latter variable will be useful just shortly, in relation to the gaze cascade effect).

Does attention relate to perceived importance of the arguments? We first ran a mixed effects linear regression analysis with signed average pros, cons difference as the dependent variable, participants as a random effect, (modeled only with intercepts; best model identified using −2 log likelihood), and scenario and AOI_{pros} as fixed effects. Both fixed effects were significant. For scenario and AOI_{pros}, we respectively observed $F(2, 135) = 3.31$, $p = .04$ and $F(1,135) = 50.7$, $p < .0005$. Next, the same regression analysis was run replacing AOI_{pros} with AOI_{pros3}. The best model was equivalent (no slopes for random effects, no two-way interactions for fixed effects). Both fixed effects were significant, for scenario $F(2,135) = 4.77$, $p = .01$ and for AOI_{pros3} $F(1,135) = 49.6$, $p < .0005$. The uncorrected correlations between signed average pros, cons and AOI_{pros} and AOI_{pros3} were respectively 0.57 and 0.55. This result shows that attention was, on average, directed towards the AOIs with the arguments which were judged more strongly – both in the last three seconds and throughout the decision period.

Next step we tested whether attentional dynamics could predict decisions and whether a gaze cascade effect occurred. The gaze cascade effect occurs when the chosen option is the one that is attended to prior to the choice. So, we looked at fixations in two time windows: first, in a time window close to the decision (the last three seconds prior to the decision); second, in a window corresponding to the rest of the time period. If we find that the eventual decision correlates more highly with fixations in the window close to the decision, we can conclude some evidence for the gaze cascade effect.

To this end, we employed mixed effects binary logistic regressions with decisions as the dependent variables. All three models included participants as random effects (no slopes, best model identified with BIC for the model with AOI_{pros} and, for consistency, we utilized the same structure in the other two cases) and scenario as a fixed effect. The models differed in terms of whether an additional fixed effect was AOI_{pros}, AOI_{pros3}, or $AOI_{pros(start\text{-}3)}$. In all models we included the two-way interaction between the two fixed effects. In all three models, the only significant effect involved the AOI variables. In the first model, with AOI_{pros}, we observed $F(1,129) = 28.07$, $p < .0005$; in the second model, with AOI_{pros3}, $F(1,129) = 33.69$, $p < .0005$; and in the third model, with $AOI_{pros(start\text{-}3)}$, $F(1,129) = 19.93$, $p < .0005$. The uncorrected correlations between (first) decision and AOI_{pros}, AOI_{pros3}, and $AOI_{pros(start\text{-}3)}$ were, respectively, -0.65 and -0.70, and -0.51 (in all cases, higher fixations for pros indicate higher probability of a yes decision, as expected; recall, yes and no were respectively coded with a 1 and 2). Note that the correlation between fixations in the last three seconds and decision is higher than for fixations in the decision period from start up to three seconds before the decision – as noted, we take this to indicate the presence of a gaze cascade effect in our data (Shimojo et al., 2003). The correlation between decision and the AOI_{pros} variable is consistent with results showing that the more participants look at a particular response option, the more likely that they will adopt it (Jahn & Braatz, 2014; Scholz, Krems, & Jahn, 2017).

Overall, the choices and eye-tracking data are in line with previous work but also cast some doubt on our attempt to increase the absolute number of between option transitions by using a more complex and ambiguous task.

3.1. Gradually quenched oscillations

The implication from the OQS4 and OQS6 models that, sometimes, there may be gradually quenched oscillations in the attentional dynamics does not map well onto any of the traditional variables and analyses. Note, this implication does not concern the number of oscillations. The quantum model is consistent with both many and few or no oscillations, even across long decision periods. The main point is that, if there are oscillations, these would have progressively lower amplitudes. It is not possible for the quantum model to produce a pattern of, for example, gradually increasing oscillations. Behaviourally, a datapoint computed for, e.g., the pros curve reflects the proportion of AOI fixations vs. everything else — attentional focus. So, if there are oscillations in the pros curve this means that attentional focus is changing from time bin to bin. Stabilisation would mean that either there are no oscillations or that the amplitude of these oscillations is small, that is, the changes in attentional focus from time bin to time bin would be small. For example, if we have stabilisation in the pros curve at say around 0.7, this means that from time bin to time bin about 70% of attention would be focussed to the pros AOI and 30% to either cons or neither. In a way, we can consider stabilisation to reflect a fixed policy for how attention is allocated to the pros or cons AOIs, a steady state in the relative importance, or at any rate the 'attention grabbing properties', of one AOI vs the other. We might further infer that such a steady state indicates fixed attentional propensities towards the different options. Note, across several decisions, we think it is unlikely that this pattern of gradually quenched oscillations would be

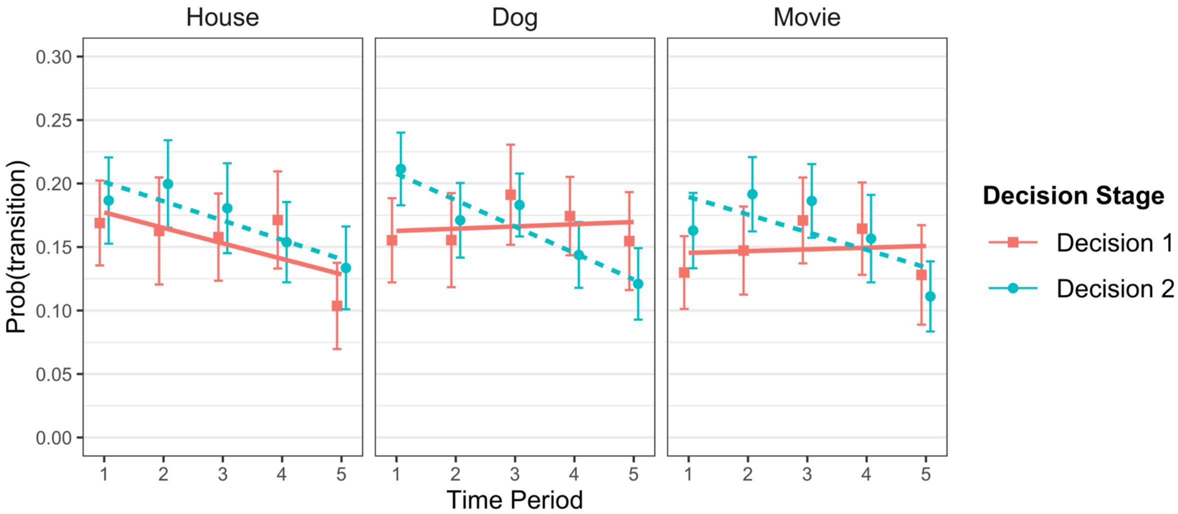

Fig. 4. Mean probability of transitions in each of the five time periods, for the three scenarios and the two decisions. Error bars show 95% within-subjects confidence intervals around the means (Morey, 2008). Lines represent smoothing with linear regression.

reflected in *all* decisions.

What are empirical measures suitable for examining this characteristic pattern from the quantum model? We cannot use the way the number of transitions between options changes across the decision period, because this would depend on the length of the period as well. The well-established SI (Payne et al., 1988) is also unsuitable, because the SI cannot reveal if transitions are gradually quenched, just the relative contribution of within and between option transitions. Additionally, the SI is not a very sensitive measure, if computed over time, because it results in missing values, if within a time bin a participant neither showed within nor between option transitions (most likely, in these cases participants kept on looking at only one pro or con argument).

The idea of stabilization is best empirically examined via the absolute value of the first derivative of the pros and cons eye tracking curves. The first derivative of a function offers a measure of the rate of change of the function. So, the first derivative at different points tells us whether at that point the function is changing violently/ steeply vs. weakly. In trying to capture the idea of gradually quenched oscillations, a distinction between violent vs. weak oscillations is helpful because it informs whether oscillations are strong vs. weak. Note, using the *absolute* value of the first derivative is necessary, since we are not interested in whether the function is increasing or decreasing, at a particular point. So, we first computed a numerical approximation of the first derivative for pros and cons curves for each decision separately (recall, per participant there are six decisions: three scenarios times two decisions for each scenario), by taking the absolute value of the differences (of the pros or cons) between successive points and averaging them in each of five equally spaced (for each decision) time periods. The test of the quantum model implication that there are gradually quenched oscillations rests in the extent to which, across these five time periods, the average of the absolute derivative values is reduced.

A less direct empirical test concerns the probability of a transition (within and between). Such probabilities can be computed for a particular time period in a decision, by counting the number of transitions and dividing them by the number of time bins in the time period. It might be possible to have quenched oscillations/ stabilisation with many transitions, if the rate of these transitions is such that it does not change the relative proportions of attentional allocation. However, this seems unlikely and we suggest that higher probability of transitions would be indicative of lower stabilisation.

We examined these two dependent variables, probability of transitions and absolute derivatives (as above), in mixed effects linear regression models, with participants as a random effect, and three fixed effects: scenario (house, dog, movie), decision stage (one or two), and time period (1–,5). The random effect was modelled with intercepts only. Following the omnibus model, for each of the six decisions separately, we conducted a linear trend analysis, to examine whether the mean of the dependent variable in each of the time periods progressively decreased. We ran these contrast analyses regardless of the significance of the three-way interaction between scenario, decision, and time period, because each decision can be treated independently and, for each decision, of paramount interest is exactly this linear trend analysis for the time period fixed effect. It is this analysis which will reveal whether, across decisions, there is any evidence for a pattern of quenched oscillations across the five time periods. This approach was repeated for the two dependent variables, probability of transitions and absolute derivatives (employing the R, R Core Team, 2020, packages lme4, Bates, Maechler, Bolker, & Walker, 2015, afex, Singmann, Bolker, Westfall, Aust, & Ben-Shachar, 2021, and emmeans, Lenth, 2021). The code for these regression models can be found in the OSF page for the project (https://osf.io/vpdx5/).

Regarding probability of transitions, we employed linear mixed models, which were fitted using residual maximum likelihood estimation. Fixed effects were evaluated via the Satterthwaite approximation of degrees of freedom and sum-of-squares contrast coding. Results are shown in Fig. 4. We found a main effect for time period, $F(4,1276) = 9.63$, $p < .001$), and a significant interaction between time period and decision stage, $F(4,1276) = 3.12$, $p = .014$). There were no main effects for scenario, $F(2,1276) = 1.18$, $p = .307$, or decision stage, $F(1,1276) = 3.03$, $p = .082$, or a significant three-way interaction between scenario, decision stage, and time period, $F(8,1276) = 0.75$, $p = .647$. Linear trend analyses revealed a significant decrease in transition probabilities over time for four

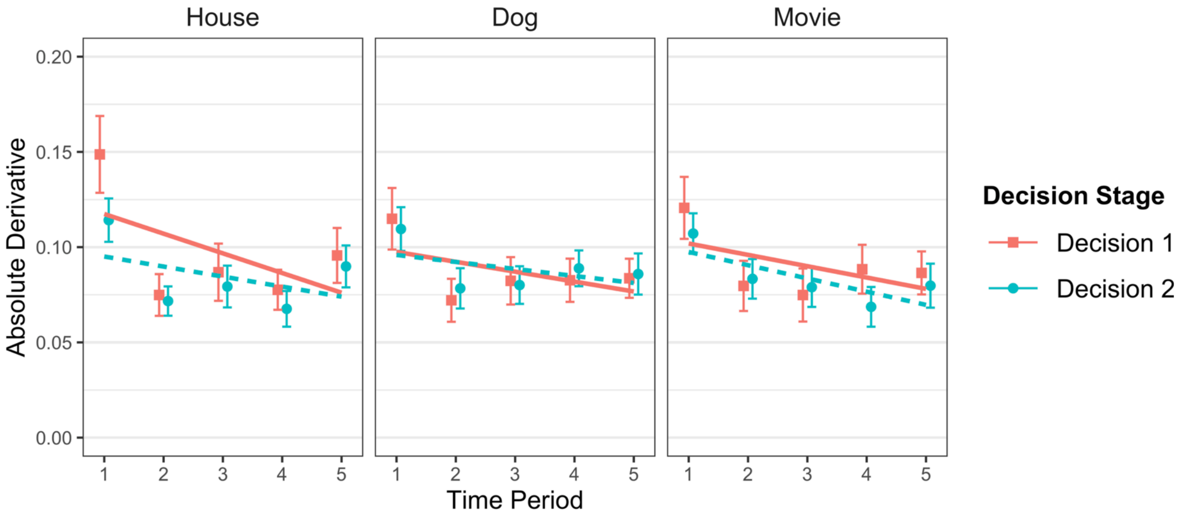

Fig. 5. Mean absolute first derivative values in each of the five time periods, for the three scenarios and the two decisions. Error bars show 95% within-subjects confidence intervals around the means (Morey, 2008). Lines represent smoothing with linear regression.

out of six decisions. For the first decision in the house, movie, dog scenarios and the second decision in the scenarios in the same order, we observed, respectively, $p = .0195$, $p = .7354$, $p = .7947$, $p = .0036$, $p = .0001$, $p = .0079$.

We employed the same analyses for derivatives. Results are shown in Fig. 5. The main effects of time period and decision stage were both significant (respectively, $F(1,1276) = 51.44$, $p < .001$ and $F(1,1276) = 6.80$, $p = .009$), but not scenario ($F(2,1276) = 1.10$, $p = .33$). Furthermore, we found significant interactions between time period and scenario, $F(2,1276) = 2.65$, $p = .007$, time period and decision stage, $F(4,1276) = 2.42$, $p = .046$, and decision stage and scenario, $F(2,1276) = 3.24$, $p = .04$. There was, however, no significant three-way interaction between scenario, decision stage, and time period, $F(8,1276) = 0.85$, $p = .556$. The linear trend analysis for each of the decisions produced significant results for five out of the six decisions. For the first decision in the house, movie, dog scenarios and the second decision in the scenarios in the same order, we observed, respectively, $p < .0001$, $p = .0063$, $p = .0018$, $p = .0057$, $p = .0536$, $p = .0003$. These results generally confirm that, in several cases, there is a pattern of quenched oscillations, as evidenced by both a reduction in the probability of transitions and a reduction in the absolute values of the first derivative.

Notwithstanding this fairly positive impression, a few qualifications are in order. First, in most cases we seem to observe a rapid decline in oscillations that level out quickly, rather than a more gradual decline. Empirically, we observed fewer oscillations than we expected. It is possible that, with tasks with more oscillations (e.g., for harder or more important decisions), we would find a more gradual decline. The OQS model is consistent with both a pattern of gradual and a pattern of rapid decline, but, in its current form, cannot predict which specific pattern we would observe. Second, there is a question of whether there might be an alternative, more sensitive method for examining the data. Specifically, our discussion around oscillations might tempt a suggestion that a Fourier/frequency analysis would be suitable. However, there are a few problems, notably it is unclear how a Fourier analysis can apply to a short, asymmetrical signal. In signal analysis there are techniques for 'extrapolating' a limited-time signal to infinite time (e.g., upsampling techniques), but it is an open, complex question how such techniques could apply to data on attentional dynamics. Moreover, for many data curves we observed stabilization in part of the range. Applying a Fourier analysis to straight lines is tricky and dependent on auxiliary assumptions. Overall, there is no straightforward way to apply a Fourier analysis to the present data. Third, and finally, one might say that the analyses in this section are not sufficient to test the predictions of quantum models in all respects. The idea of gradually quenched oscillations is our attempt to provide a broad level concept, to approximately describe the function of the quantum models. However, ultimately, for the purpose of testing a complex computational model, there is no substitute to just fitting the model.

4. Formal modeling

The main purpose of this work is to evaluate whether the OQS4 or OQS6 models can fit the attentional dynamics curves for the first and second decision stages. We will also consider whether model parameters can reveal subtler structure in eye tracking dynamics, in terms of predictive capacity for decisions produced.

We fitted the eye tracking data from each participant and for each scenario separately. For each participant there were three curves for each scenario, one corresponding to fixations within the AOI for the pros and the yes response button, one for the cons and the no response button, and one for neither. The third curve would be an agglomeration of white space fixations, eye blinks and saccades. Blink rates would be influenced by several factors, such as fatigue (including offloading perceptual input to aid concentration) and dryness of eyes. Because of these ambiguities, we focused model fitting on the pros and cons curves.

The raw eye tracking curves (time bin 100 ms) appear highly erratic and exponential smoothing was applied to remove high frequency noise, as is common practice in time series analysis. Given raw data $d(t)$, then $x(t) = \alpha \cdot d(t) + (1 - \alpha) \cdot x(t-1))$, using $\alpha =$

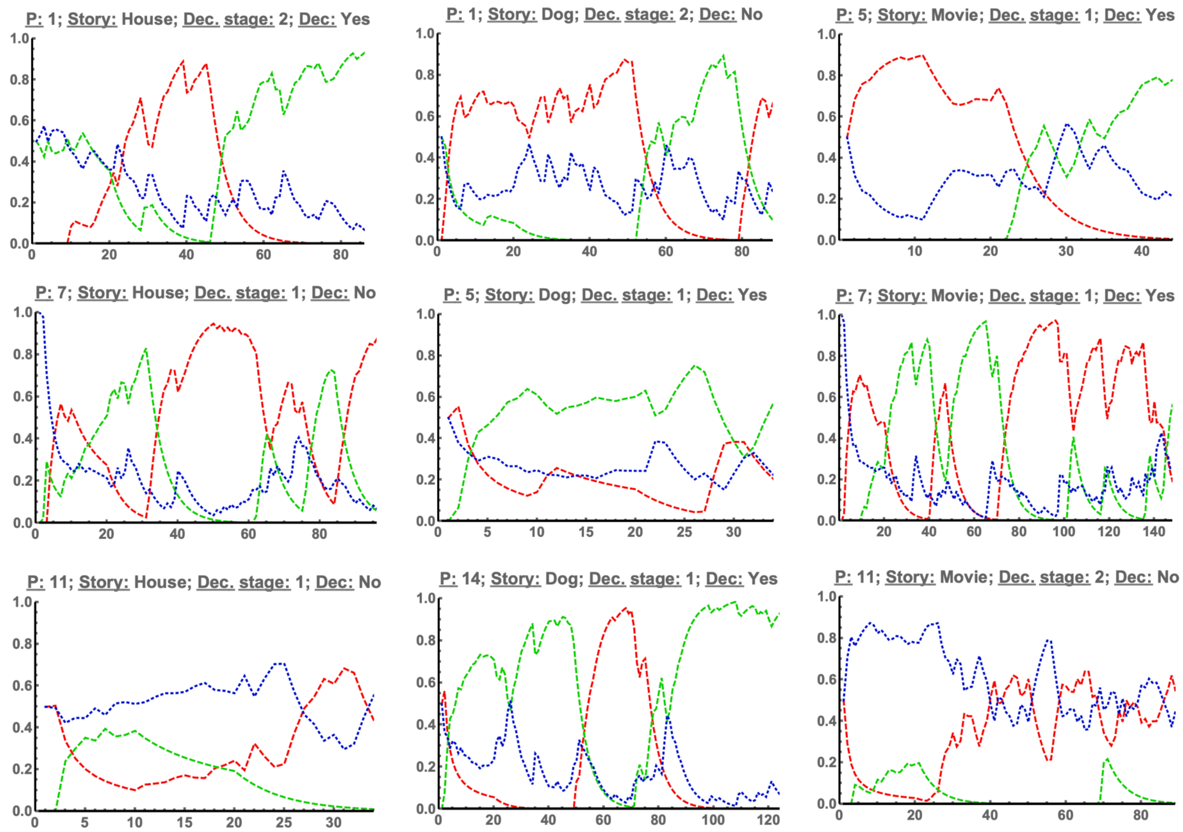

Fig. 6. Examples of eye tracing curves for the House (left), Dog (middle), and Movie (right) scenarios. The green line corresponds to fixations in the pros/ yes AOI, the red line to fixations in the cons/ no AOI, and the blue dotted line to neither (which includes eye blinks). The horizontal line is time bin, corresponding to 200 ms; note, second stage decisions end shortly after 75 time bins (15 s). The vertical line indicates fixation proportion.

0.1. Values of α closer to 0 and 1 have a lower and higher smoothing effect respectively. Higher α means that changes in the smoothed data lag behind true values to a greater extent, i.e., the exponentially weighted average is an estimate of where the true level was $1/a$ time points ago relative to the latest data point (Nau personal communication). This approach to smoothing works as follows. For example, for the first point $x(1) = d(1)$, the second point is approximately the average between $d(1)$ and $d(2)$, the third one $x(3)$ is a combination of $d(1)$, $d(2)$, and $d(3)$ with more recent points having greater weight and so on. We also initialized $x(0) = 0$ and corrected for the initialization bias as $x_{corrected}(t) = \frac{x(t)}{1-(1-a)^t}$, see Section 3 of Kingma and Lei Ba, 2015). Following smoothing of data at 100 ms bins, we averaged consecutive bins to create 200 ms bins, as this time scale was considered more appropriate for model fitting.

We employed three models, a null model of two mean-centered straight lines, OQS4, and OQS6. Note, the straight lines model is particularly apt here, since straight lines would produce reasonable fit for broadly oscillatory patterns. The null model consisted of one line for the pros curve and one line for the cons one. Briefly summarizing, OQS4 involved a Hamiltonian for intrinsic oscillation and two processes of weight transfer (drifts) from the undecided state to either the yes or the no states. OQS6 was as for OQS4 but included two additional processes of weight transfer from either certain state to the undecided state. For both decisions, the initial state was set to undecided ($\rho(t = 0)$ such that the only non-zero element was $\rho_{22} = 1$). From each quantum model we extracted a curve for pros and one for cons. Note, even though the initial state is classical (relative to the canonical basis for this problem), non-classical effects can still arise (Busemeyer & Bruza, 2011; Khrennikov, 2016).

Model evaluation was based on residual sum of squares (RSS) per participant per scenario, computed for the pros and cons curves and aggregated. The three models are approximately nested, with OQS4 and OQS6 obviously so. The straight lines model can be approximately recovered from the quantum models. Therefore, model comparison was based on the Bayesian Information Criterion (BIC; Schwarz, 1978), computed for residual sum of squares (RSS), as $BIC = n \cdot \ln\left(\frac{RSS}{n}\right) + k \cdot \ln(n)$, where n are the data points (time bins) and k the number of parameters (OQS6, OQS4, and the null models had 6, 4, and 2 parameters respectively). Given the total number of data points for the first decision, 9,987, and for the second, 11,743, BIC values for the first and second decision are broadly comparable, with some caution.

Model fitting was performed in Mathematica. First, numerical, parametrized solutions were obtained for Equations (1), (2), using Mathematica's differential equation solver. Second, best parameters were obtained using Mathematica's nonlinear model fitting function. Model fitting was computationally intensive, with fits of each model requiring approximately three weeks of computation

Table 2
Summary of model fits for the first and second decision in the experiment. When comparing OQS4, OQS6 with the null models a positive value means that the null model is inferior. When comparing OQS4 with OQS6 the negative values mean that the former is superior. "Instances of better fit" refers to the number of times out of a total of 135, for which the BIC was better for the OQS models than for the null model or (in the last column) the OQS6 model compared to the OQS4 model.

Variable	null BIC - QOSyD4 BIC	null BIC - QOSyD6 BIC	6 param. relative to 4 param.
Decision 1, Bin 200 ms			
Instances of better fit (BIC)	98	60	10
Aggregate BIC	1599.37	281.84	−1317.53
Decision 2, Bin 200 ms			
Instances of better fit (BIC)	104	70	39
Aggregate BIC	2621.46	914.36	−1707.10

time on an eight-core Mac Pro.

Finally, Fig. 6 shows some examples of (smoothed) eye tracking curves. Without smoothing, attentional dynamics looks random, with a very large number of oscillations. Even in Fig. 6, the dynamics often looks erratic. Is there a pattern of gradually quenched oscillations? Just on the basis of visual inspection, it is impossible to ascertain such a claim – the empirical case is based on the analysis employing probability of transitions and absolute values from the first derivative. Even though the linear trends supported our expectation, these are not clear cut to the point that they can be visually apparent. Ultimately, the quantum models are constrained so that, if there are any oscillations at all, these should either persevere indefinitely or gradually diminish, and so the success of the model fits, or not, is the most direct test of these ideas. Note, in Fig. 6 transitions would be indicated by a change from a focus on one AOI to another – but because Fig. 6 shows smoothed data, visual impression about transitions from Fig. 6 cannot be mapped to the number of transitions we have in Table 1.

A summary of fit results is shown in Table 2, Appendix E presents detailed results (per participant), and Appendix F some examples of (smoothed) actual data vs. fit curves (the full list of these graphs is available in the OSF page for the project: https://osf.io/vpdx5/). We considered aggregate BIC values as well as the number of instances when OQS4 or OQS6 are associated with BIC values (even just) superior to those for the null models. Note, there are 135 such instances for each decision (45 participants times 3 scenarios per participant). We can clearly conclude that OQS4 is superior to the null model for both the first and the second decision. In the second decision we anticipated a cognitive process of reevaluation of the available evidence, implemented in OQS6 as drifts from certainty to uncertainty, but there was no evidence for this. We conclude that the additional flexibility in OQS6 did not translate to better fit in either the first or the second decision, compared to OQS4 (but OQS6 was still better than the null model). A cautionary note concerns the ability of the optimization process to identify sufficiently good solutions.

One interesting question is whether the characterization of eye tracking dynamics with the OQS4 can inform the eventual decision. The model parameters interact with each other so it is not possible to establish a simple association between a model parameter and a behavioral variable. Focusing on the yes decision, the model process pushing stabilization towards yes, that is $\rho_{11}(largetime) = 1$, is transfer of amplitude from uncertainty to yes and this process conflicts with the one pushing stabilization towards no, that is, $\rho_{33}(largetime) = 1$. However, as the d parameter increases relative to the a_{ij} parameters, ambivalence in the system dominates trends towards yes or no and stabilization typically occurs at $\rho_{11}(largetime) = \rho_{33}(largetime) = 0.5$. We therefore explored whether this function of parameters can be related to decisions, $(a_{12} - a_{32}) \cdot e^{-Abs(d)}$, as a simple function of the difference between the two drift parameters towards yes vs. no responses, multiplied by a factor which 'squashes' differences when the absolute value of d is large.

To assess this possibility, we employed a mixed effects binary logistic regression model, with decision as the dependent variable, participants as a random effects (modeled only with intercepts), decision phase and scenario as fixed effects categorical independent variables, and the function $(a_{12} - a_{32}) \cdot e^{-Abs(d)}$ as a fixed effects continuous independent variable (no interaction terms were included). There was a significant effect for the parameter function, $F(1, 256) = 19.99, p < .0005$; the uncorrected correlation was -0.25 (this is in the expected direction). Excluding the 'squashing factor', the uncorrected correlation was -0.24. We think the finding that model parameters have some (modest) predictive value regarding the eventual decisions is intriguing, given the apparent complexity of the eye tracking curves. Note, there was also a significant effect of scenario, $F(2, 265) = 15.26, p < .0005$, but no effect for decision phase.

5. General discussion

Bayesian theory and drift diffusion models represent two of the most important standards for modeling probabilistic inference and dynamics in decision making (for alternative approaches see e.g., Glöckner & Betsch, 2008; Glöckner et al., 2014; Jekel et al., 2018). Beyond these frameworks, there are several options for cognitive modelling, some of which unexplored and rife with explanatory opportunity. In this work we pursued one theoretical innovation regarding the understanding of dynamical changes in attentional focus, in an ambivalent decision task: we explored the possibility that the dynamics reflect a competition between the degree of intrinsic oscillation vs. drift processes towards to (or away from) the various options. Intrinsic oscillations could be, conceivably, incorporated in standard diffusion theory. However, this would require post hoc adjustments. By contrast, in quantum theory there is already a well-developed technical framework for dynamical change, which balances drifts towards particular options vs. intrinsic oscillation. In physics, this framework of OQS concerns the way a microscopic system interacts with the environment: left by itself, a microscopic system can retain quantum character indefinitely and display unquenched oscillatory behavior (for some observable); when interacting with its environment, quantum character and oscillatory behavior are gradually diminished, and the system settles onto a particular (non-quantum) state. There are intriguing links with psychological theory, notably the ideas of Fodor (1983) regarding the degree to which a cognitive inference is influenced by other information in a person's knowledge base.

We developed two quantum models, such that the first, OQS4, incorporated two drift processes, one from uncertainty to a yes response and another to a no response, and the second, OQS6, included in addition drift processes from the two certainty states (yes, no) to uncertainty. In our experimental task participants had to make two decisions. Possibly, for the first decision participant data would reflect just drifts from uncertainty to certainty, but for the second decision there might be drifts from certainty to uncertainty too, as participants were called to reevaluate their second opinion. This expectation was not confirmed and the OQS4 model was superior to the OQS6 one for both decisions (both models were superior to the null for both decisions). This constitutes the main way in which we evaluated the proposed models and complements other recent work to understand eye tracking data (Tatler et al., 2017). The reasonably good fits especially for OQS4 offer promise in the idea that there are instances of cognitive process where it is reasonable to postulate intrinsic oscillation, which competes with drifts towards possible responses. Also, even though there was no evidence for OQS6 over and above OQS4 in the present work, we consider appealing the characteristic of the present formalism which allows different drifts towards particular combinations of options. It is tempting to think that for instance a drift away from a state cannot be differentiated from a drift towards a state, but in the complex formalism of OQS this is not the case (cf. the examples in Fig. 1). With future work we hope to develop decision paradigms which offer more opportunity to tease apart such differences in drift processes.

We think the present empirical approach is reasonably defensible, as an instance of decision making which extends the more typical matched-attribute, simpler decision problems. It seems intuitive that if we are to study ambivalence in attentional dynamics, we should be employing decision problems with neither obvious answers nor simple heuristics to produce such an answer. However, the empirical paradigm suffered from various assumptions which were exploratory, lacking previous studies on ambivalence which could offer guidance. An unavoidable limitation has been separating the processing of the information from the decision process. We are unsatisfied with this separation, but also lacked insight for what could have been a preferable alternative procedure. Another limitation is that our data and analyses concern attentional dynamics, but an alternative interest is decision dynamics. A challenge for future work is to develop paradigms which allow measurement of confidence towards different decision options, without directly asking participants – this is not straightforward, because there is extensive evidence that intermediate decisions can impact on the mental states in a way that interferes with subsequent ones (e.g., Sharot et al., 2010; White et al., 2020), though perhaps indirect methods of measuring decisional propensities, such as EEG, might work (cf. Kohl et al., 2020).

Another major challenge for future work is to explore more comprehensively alternative pathways for modelling such data. One important caveat is that our work does not allow ruling out plausible alternative models. For example, considering the low frequency of between option transitions of about three, the data could also be generated by the following simple search process: individuals search information mainly within options; they switch between the options once to read both of them; they then conduct a second comparison to double check the conclusion (going once more back and forth). This would be in line with what was expected by one of the authors (AG) based on previous evidence and theory (i.e., the parallel constraint satisfaction theory of decision making; Glöckner & Betsch, 2008; Glöckner & Herbold, 2011). A higher number of transitions would have ruled out such a strategy and therefore would have provided more evidence for the proposed quantum mechanism. Note, however, that such models would not necessarily compete with a probabilistic model, e.g., based on quantum theory. In some cases, mimicries in cognitive science can refine interpretation of a probabilistic model, without invalidating the probabilistic model (cf. Kellen, Singmann, & Batchelder, 2017, who offered a heuristics-based model for some predictions from quantum cognitive models). Also, some alternative explanations can be ruled out with the current data. Given the high intercorrelation between the two choices for the same task, it is unlikely that persons reached an equilibrium of being indifferent between the options and just showed random switching during information search. Also, the application of strict non-compensatory heuristics (e.g., take the best heuristic, Gigerenzer & Goldstein, 1996) or a strict computational weighted additive strategy (e.g., Payne et al., 1988) seem to be unlikely, given that we observe way too many within option transitions for non-compensatory heuristics (which would predict zero transitions in most cases) and somewhat too many between-option transitions for a strict weighted additive strategy (which would predict only one). However, an in-depth classification of individual strategies would be required to test this conclusion further.

Overall, to be clear, we think our conclusions regarding the OQS4 model offer reasonable support for this approach, but this impression needs to be moderated by the inevitably exploratory nature of the present research. A comparison with alternative models (e.g., Glöckner & Herbold, 2011; Gluth et al., 2020), although potentially interesting, would go beyond the scope of this paper. We leave it to future research to develop methods for fitting these models to the data and testing them against the OQS approach.

We hope that the modeling framework on which OQS4 and OQS6 are based will continue to be developed. It may be appealing to consider whether such models can be augmented with a memory store, though there is no obvious route for doing so and, in addition, the way the state develops with time already captures the information processing taken place so far. A more obvious direction is to explore the available options for dynamical processing, going beyond the fairly minimal specification we adopted in this work. One could look to alter the Hamiltonian in the model. Currently, we have adopted a simple Hamiltonian, which just embodies a process of weight transfer from the uncertainty state to each of the certainty states, and back. This Hamiltonian could be elaborated, for example, allowing the weights towards yes vs. no to be differentiated (Pothos & Busemeyer, 2009; Trueblood & Busemeyer, 2011). However, note that parameters in the Hamiltonian do not translate to biases for decisions in a straightforward way and Schrödinger's equation by itself will in general produce indefinitely oscillatory behavior (Broekaert et al., 2017).

A more novel direction offered by the present work considers the C operators, which could be specified to reflect more complex ways in which drifts can occur. For example, in the current two models we only consider drifts from uncertainty to either of the certainty states and vice versa. We could introduce a drift directly from one certainty state to the other (analogous to lateral inhibition in some sequential sampling models, e.g., Johnson & Busemeyer, 2005) or create C matrices involving a particular balance between drifts across the different options (currently, each C matrix incorporates drift from a single state to another state). Relatedly, a quantum model can be set up with several distinct outcomes and there is potential to employ this flexibility with a view to explore the range of psychological states which might be outcomes from an ambivalent decision process.

We have endeavored to develop the present models by analogy to drift diffusion ones, insofar that there are distinct parameters embodying drifts towards particular options. An alternative approach would be to more directly implement a sequential sampling process using quantum theory. For example, Busemeyer, Wang, and Lambert-Mogiliansky (2009), Kvam et al. (2015) have proposed 'quantum random walk' models, using a tri-diagonal Hamiltonian, which evolves amplitude across an entire ratings scale. These models have offered many valuable insights. Both the present model and these quantum random walk models are based on similar principles. But, as with sequential sampling models in general, it is a little unclear how quantum random walk models could be applied to the present data. Also, we think there is merit in further considering the balance between intrinsic ambivalence (oscillation) vs. drifts towards different options; this idea is the main theoretical contribution from the present work.

Finally, it is well known that eye tracking structure can reveal a person's eventual decision, that is, the gaze cascade effect (Krajbich et al., 2012; Shimojo et al., 2003). In the present data we also observed a gaze cascade effect. We were furthermore surprised that a simple function of the model parameters correlated, albeit very weakly, with the eventual decision. Exploring the way eye tracking curves can be mathematically described thus shows potential for structure in such curves that can be related to decisions. The intriguing possibility is that attentional dynamics may impact on decisions more so than previously thought (Shimojo et al., 2003; Suri & Gross, 2015).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Author contribution

EMP, IB, AR, BvH, and AG conceptualized the study. AR did the investigation and data curation. The formal analyses were done by IB, EMP and AR. All authors contributed to the Methodology. EMP and AR drafted the original version of the manuscript and revisions. All authors provided feedback.

Appendix A

Some additional mathematical illustrations

Recall from main text the form of the Lindblad equation:

$$\dot{\rho} = -i[H,\rho] + \sum_j \Gamma_j \left(C_j \rho C_j^\dagger - \frac{1}{2}\rho C_j^\dagger C_j - \frac{1}{2} C_j^\dagger C_j \rho \right)$$

We can further illustrate the workings of the Lindblad equation. The first term in parenthesis is responsible for quantum jumps and the other two terms (prefixed by one half) normalization terms. We can illustrate their impact by considering a simplified form for the Lindblad equation as follows:

$$\begin{aligned}\dot{\rho} &= -i[H,\rho] + \Gamma\left(C\rho C^{\dagger} - \frac{1}{2}\rho C^{\dagger}C - \frac{1}{2}C^{\dagger}C\rho\right)\\ &= -i\left(H\rho - \rho H - i\frac{\Gamma}{2}\rho C^{\dagger}C - i\frac{\Gamma}{2}C^{\dagger}C\rho\right) + \Gamma C\rho C^{\dagger}\\ &= -i\left(H\rho - \rho H - i\frac{\Gamma}{2}\rho C^{\dagger}C - i\frac{\Gamma}{2}C^{\dagger}C\rho\right) + \Gamma C\rho C^{\dagger}\\ &= -i\left(\left(H - i\frac{\Gamma}{2}C^{\dagger}C\right)\rho - \rho\left(H - i\frac{\Gamma}{2}C^{\dagger}C\right)\right) + \Gamma C\rho C^{\dagger}\\ &= -i\left(H_{eff}\rho - \rho H_{eff}^{\dagger}\right) + \Gamma C\rho C^{\dagger}\end{aligned}$$

where $H_{eff} = H - i\frac{\Gamma}{2}C^{\dagger}C$ (note this is not a Hamiltonian operator, since it is not self-adjoint, but it can be thought of as approximately so). Noting that any density matrix can be written as $\rho = \sum_j p_j|\psi_j\rangle\langle\psi_j|$, we have $\dot{\rho} = \sum_j p_j\left[-i\left(H_{eff}|\psi_j\rangle\langle\psi_j| - |\psi_j\rangle\langle\psi_j|H_{eff}^{\dagger}\right) + \Gamma C|\psi_j\rangle\langle\psi_j|C^{\dagger}\right]$.

This equation provides insight into the structure of the Lindblad equation. We can see that each of the pure state components can have an approximately Schrödinger evolution as $|\dot{\psi}_j\rangle = -iH_{eff}|\psi_j\rangle$, but also each component can change by quantum jumps, so that $|\psi_j\rangle \to C|\psi_j\rangle = |\psi_k\rangle$. This can also be illustrated by introducing a small time period δt. After δt, if there were no jumps, we would have $|\psi_j(t+\delta t)\rangle = \frac{(1-iH_{eff})|\psi_j\rangle}{\sqrt{1-\delta p_j}}$, where we have introduced $\delta p_j = \delta t\Gamma\langle\psi_j|C^{\dagger}C|\psi_j\rangle$. With a jump, we instead have $|\psi_j(t+\delta t)\rangle = \sqrt{\frac{\Gamma\delta t}{\delta p_j}}C|\psi_j\rangle$. That is, with probability $1-\delta p_j$ the system evolves due to H_{eff} and with probability δp_j it jumps to another state. Note, because of linearity, this picture based on individual pure states $|\psi_j\rangle$ scales up to that for a density matrix.

The final consideration is to understand the quantum jumps, since it is these elements which correspond to the interaction between target system and its environment and allow stabilization. As noted in main text, particular C operators in isolation can be thought of as driving stabilization towards particular basis vectors (eigenstates of the question/ observable of interest). As an example, let us consider $C_{13} = \begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$ and $|\psi\rangle = \begin{pmatrix}x\\y\\z\end{pmatrix}$. Then, $C_{13}|\psi\rangle = \begin{pmatrix}z\\0\\0\end{pmatrix}$, which is the first eigenstate (without normalization, which we are ignoring here for illustration).

$$C_{12} = \begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}, C_{12}|\psi\rangle = \begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}y\\0\\0\end{pmatrix}$$

$$C_{32} = \begin{pmatrix}0&0&0\\0&0&0\\0&1&0\end{pmatrix}, C_{12}|\psi\rangle = \begin{pmatrix}0&0&0\\0&0&0\\0&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\y\end{pmatrix}$$

Thus, each C term embodies drift towards particular options, and allows us to motivate the particular form of the OQS4 and OQS6 models we adopted.

Using this picture of quantum jumps, we can say that a Lindblad term such as $a_{12}C_{12} + a_{32}C_{32}$ reflects an assumption that the decision process involves jumps from the undecided state ρ_{22} to each of the two definite answer states, ρ_{11} and ρ_{33}, and stabilization will have some bias to occur towards yes (ρ_{11}) or no (ρ_{33}) responses – this is the motivation for the OQS4 model.

We next provide some illustrations for model behavior, for OQS6, Fig. A1. In all cases the vertical shows $\rho_{11} - \rho_{33}$ at large times. The left figure illustrates the way dominance for yes response requires both drift for a yes response (a_{12}) and drift away from a no response (a_{23}), even if the former influence is more important, with other parameters set as $d = 0.1$, $a_{32} = a_{21} = 1$ and $adjT = 1$ (the latter in all cases). The middle figure shows how drift for a yes response (a_{12}) balances out drift for a no response (a_{32}), with other parameters as $d = a_{23} = a_{21} = 0.1$. The right illustrates that high values of d prevent strong dominance of (e.g.) yes response; other parameters $a_{32} = a_{21} = 0, a_{23} = 5$.

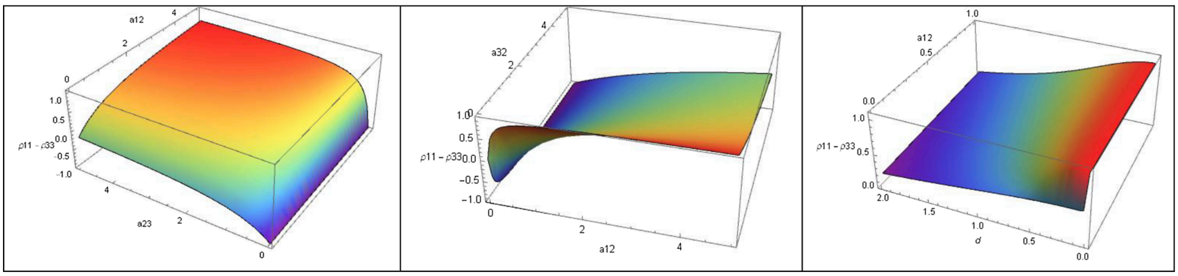

Fig. A1. Illustrating OQS6.

Appendix B

Pretest to balance the pros and cons in each scenario

We created three scenarios corresponding to everyday life decisions. For each scenario, thinking like the protagonists, participants had to make a yes vs. no answer regarding a dilemma, based on pros and cons whose importance could not be precisely quantified. We ran a brief pretest experiment, to identify pros and cons that were broadly well balanced.

B.1. Participants

Participants were 36 psychology students at City, University of London, who received course credit for participation (mean age 18.78; three males). All participants were experimentally naïve and had normal or corrected to normal vision.

B.2. Materials and procedure

Participants were presented with three decision scenarios, such that all questions for one scenario would have to be resolved, prior to presenting any information for the next scenario. In all cases, a young couple was faced with a binary decision, to buy a house, to adopt a stray dog, to go to a movie. Participants first read the story and then, in a subsequent screen, were presented with a list of 12 relevant arguments, six cons and six pros. The list was organized so that a pro always followed a con and vice versa (a fixed randomly determined order was used). For each of these factors, participants had to provide a simple rating (anchors 0, no importance at all, to 10, extremely important), indicating their perception of how important the factor was regarding the decision at stake. Following the rating of the factors, in a separate screen, participants were asked to indicate whether they thought the couple was likely to make the decision or not; this final decision was indicated as a yes or no. The task was designed in Qualtrics and lasted approximately 15 min. The task was administered online.

B.3. Results

We explored whether the proportion of yes responses was matched to those of no responses for each of the three scenarios and the average strength of the pros and cons (Table B1). For the House and the Movie stories, there was a high proportion of yes responses. So as to encourage conditions of ambivalence for each of the three stories, we discarded the most highly rated pro and the least likely rated con for the House and Movie scenarios, while for the Dog one we eliminated the most highly rated pro and con. Independent samples Bayesian t-tests, testing the hypothesis that the average strength between pros and cons was equal, provided partial evidence that the balancing improved in the Dog and Movie cases (Table B1). Note, this was an item-specific analysis, ignoring participant variability.

Appendix C shows the stories and the selection of pros, cons for each story (we included the final version of the materials, since the pros, cons selected in this pilot were further slightly adjusted to match better the Swiss sample for the main eye-tracking experiment).

Table B1
The average strength of pros and cons for the three scenarios, before and after the changes in the factors.

Story	Proportion of Yes responses	Before selection			After selection		
		Pro	Con	BF_{10}	Pro	Con	BF_{10}
House	25/36 = 69%	6.7	6.3	0.519	6.4	6.6	0.529
Dog	17/36 = 47%	5.6	6.4	0.834	5.5	6.1	0.724
Movie	28/36 = 78%	6.8	5.7	1.253	6.6	6.0	0.685

Appendix C

The three scenarios and the corresponding pros, cons, as adjusted for the main eye-tracking version of the experiment

We first show the spatial layout for the scenarios ‘going to the cinema’ and ‘keeping a stray dog’ (Fig. C1; the text for each scenario need not be read off the figures, it follows just below the figures).

House story:

Neil and Sarah are in their late twenties and have been living together for a few years. They have been looking to buy a house in the city of Zetaville where they live. Sarah has just called Neil with the news that she found a very promising two-bedroom house, near the Zetaville city centre. They have to decide whether to make an offer or not really soon.

(Neil und Sarah sind Ende zwanzig und leben seit einigen Jahren zusammen. Sie wollen in ihrem Wohnort Zetaville ein Haus kaufen. Sarah hat gerade eben Neil angerufen, um ihm zu erzählen, dass sie von einem äusserst vielversprechenden Haus mit zwei Schlafzimmern in der Nähe des Stadtzentrums erfahren hat. Sie müssen sich bald entscheiden, ob sie ein Kaufangebot machen wollen oder nicht.)

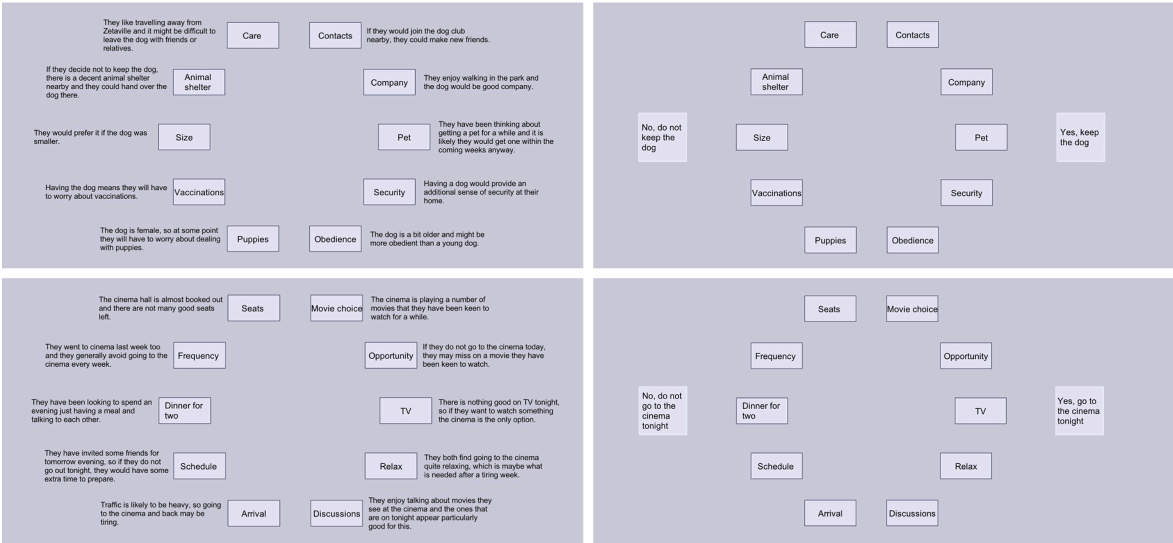

Fig. C1. Spatial layouts of scenarios Movie and Dog.

Pros	Cons
Restaurants: They enjoy going out and the house has easy access to the Zetaville city centre with its many restaurants (Restaurants: Sie genießen es, auszugehen und das Stadtzentrum von Zetaville, mit seinen vielen Restaurants, ist gut vom Haus aus zu erreichen.)	Bedrooms: They have been looking for a three bedroom house, because they want to have children soon. (Zimmer: Sie würden ein Haus mit drei Schlafzimmern vorziehen, weil sie bald Kinder haben wollen.)
Park: There is a big park close by, which would be great for leisure activities. (Park: Es gibt einen grossen Park in der Nähe, welcher sich gut für Ausflüge eigenen würde.)	Schools: The schools in the Zetaville city centre are not as good as in the suburbs. (Schule: Die Schulen im Stadtzentrum von Zetaville sind nicht so gut wie die Schulen in den Vororten.)
Neighbors: They have asked about the prospective neighbours and it looks like they would really fit in. (Nachbarn: Sie haben sich nach den zukünftigen Nachbarn erkundigt und es sieht so aus, als würden sie gut zu ihnen passen.)	Price: The house is a bit more expensive than they were planning to spend. (Preis: Das Haus ist etwas teurer als die Summe, die sie ausgeben wollen.)
Garden: The house has a garden the size they wanted, not too big or too small. (Garten: Der Garten des Hauses hat genau die Grösse, die sie gerne hätten — nicht zu gross oder zu klein.)	Redecoration: The house needs some redecorating and they would prefer to not have to do this. (Renovierung: Das Haus hat einige Renovierungen nötig und sie würden es vorziehen, wenn sie nicht renovieren müssten.)
Friends: They have many friends near the Zetaville city centre. (Freunde: Sie haben viele Freunde in der Nähe des Stadtzentrums von Zetaville.)	Street: The house is on a busy street and they do not like the noise so much. (Strasse: Das Haus ist an einer belebten Strasse und sie mögen es nicht so sehr, wenn es laut ist.)

Dog story:

Ann and Alex are in their late twenties and have been living together for a few years. They have a small house in the suburbs of Zetaville, which is close to a park. One evening, they have been walking in the park and they found a stray dog. They are considering keeping the dog, but they have to decide whether to keep the dog or not really soon.

Pros	Cons
Contacts: If they would join the dog club nearby, they could make new friends. (Kontakte: Wenn sie den nahe gelegenen Hundehalter-Club besuchen, könnten sie neue Freundschaften schliessen.)	Puppies: The dog is female, so at some point they will have to worry about dealing with puppies. (Nachwuchs: Der Hund ist weiblich, also müssen sie sich irgendwann um das Thema Welpen kümmern.)
Company: They enjoy walking in the park and the dog would be good company. (Begleitung: Sie geniessen es, im Park spazieren zu gehen und der Hund wäre ein guter Begleiter.)	Vaccinations: Having the dog means they will have to worry about vaccinations. (Impfung: Einen Hund zu haben, bedeutet auch, sich um Impfungen zu kümmern.)

(continued on next page)

(*continued*)

Pros	Cons
Pet: They have been thinking about getting a pet for a while and it is likely they would get one within the coming weeks anyway. (Haustier: Sie denken schon eine Zeit lang über ein Haustier nach. Wahrscheinlich würden sie sich in den nächsten Wochen sowieso ein Haustier anschaffen.)	Size: Having the dog means they will have to worry about vaccinations. (Grösse: Sie würden einen kleineren Hund vorziehen.)
Security: Having a dog would provide an additional sense of security at their home. (Sicherheit: Ein Hund würde ihnen zuhause ein zusätzliches Gefühl der Sicherheit geben.)	Animal shelter: If they decide not to keep the dog, there is a decent animal shelter nearby and they could hand over the dog there. (Tierheim: Wenn Sie sich entscheiden, denn Hund nicht zu behalten, gibt es in der Nähe ein gutes Tierheim, wo sie ihn abgeben könnten.)
Obedience: The dog is a bit older and might be more obedient than a young dog. (Gehorsam: Der Hund ist etwas älter und gehorcht besser als es ein junger Hund tun würde.)	Care: They like travelling away from Zetaville and it might be difficult to leave the dog with friends or relatives. (Betreuung: Sie verreisen gern und es könnte schwierig sein, den Hund bei Freunden oder Verwandten zu lassen.)

Movie story:

Bob and Beatrice are in their late twenties and have been living together for a few years. It is now Friday late afternoon. They have both had a tiring week at work, but now they are home and they are trying to decide whether to go to the cinema or stay at home. They have to decide really soon, otherwise it will be too late to go out.

Pros	Cons
Movie choice: The cinema is playing a number of movies that they have been keen to watch for a while. (Filmauswahl: Im Kino laufen Filme, die sie schon seit einiger Zeit unbedingt sehen wollen.)	Arrival: Traffic is likely to be heavy, so going to the cinema and back may be tiring. (Anreise: Wahrscheinlich wird es viel Verkehr haben. Daher könnte die Fahrt ins Kino und zurück anstrengend sein.)
Opportunity: If they do not go to the cinema today, they may miss on a movie they have been keen to watch. (Gelegenheit: Falls sie heute nicht ins Kino gehen, würden sie wahrscheinlich einen der Filme, auf den sie sehnlichst gewartet haben, verpassen.)	Schedule: They have invited some friends for tomorrow evening, so if they do not go out tonight, they would have some extra time to prepare. (Zeitplan: Morgen bekommen sie Besuch von Freunden. Wenn sie heute Abend nicht ausgehen, hätten sie mehr Zeit für die Vorbereitungen.)
TV program: There is nothing good on TV tonight, so if they want to watch something the cinema is the only option. (Fernsehen: Es läuft heute Abend nichts Gutes im Fernsehen. Wenn sie einen Film schauen wollen, müssen sie ins Kino gehen.)	Dinner for two: They have been looking to spend an evening just having a meal and talking to each other. (gemeinsames Essen: Sie möchten schon seit längerem einen Abend zu zweit verbringen, an dem sie viel Zeit für ein gemeinsames Essen und Gespräche haben.)
Relax: They both find going to the cinema quite relaxing, which is maybe what is needed after a tiring week. (Erholung: Für beide ist ein Kinobesuch erholsam, daher ist es vielleicht genau das, was sie nach einer anstrengenden Woche brauchen.)	Seats: The cinema hall is almost booked out and there are not many good seats left. (Frequenz: Sie waren bereits letzte Woche im Kino und eigentlich wollen sie nicht jede Woche ins Kino gehen.)
Discussions: They enjoy talking about movies they see at the cinema and the ones that are on tonight appear particularly good for this. (Filmdiskussion: Sie diskutieren gern über Filme, die sie gesehen haben. Die Filme, die heute Abend gezeigt werden, würden sich besonders dafür eignen.)	Care: They like travelling away from Zetaville and it might be difficult to leave the dog with friends or relatives. (Sitzplätze: Das Kino ist schon fast ausverkauft und es gibt nicht mehr so viele gute Sitzplätze.)

Appendix D

The questions employed to explore the personal relevance of each scenario and decision

1. How much have you relied on your own experiences and preferences to make the decision for the couple in this situation? (Wie sehr haben Sie sich auf Ihre eigenen Erfahrungen und Vorlieben verlassen um die Entscheidung für das Paar in dieser Situation zu treffen?)
2. How relevant is such a decision for you personally? (Wie relevant ist momentan so eine Entscheidung für Sie persönlich?)
3. How much experience do you have with this kind of decision? (Wieviel Erfahrung haben Sie mit dieser Art von Entscheidung?)
4. How much have you tried to make a good decision? (Wie sehr haben Sie sich bemüht eine gute Entscheidung zu treffen?)

Appendix E

BIC values

See Table E1

Table E1

BIC values for OQS4 and OQS6. Red shaded cells correspond to BIC differences for OQS4 or OQS6 higher than for the null model of straight lines for the pros and cons. Lower BIC values indicate better fit, so positive cell values indicate OQS model superiority to null. Green shaded cells indicate instances when the BIC for OQS6 is (even just) superior to that for OQS4.

Participant	Scenario	Decision 1			Decision 2		
		Null BIC - OQS4 BIC	Null BIC - OQS6 BIC	BIC 6 param vs. 4 param.	Null BIC - OQS4 BIC	Null BIC - OQS6 BIC	BIC 6 param vs. 4 param.
1	House	4.93	2.37	-2.56	35.43	26.33	-9.10
	Dog	6.19	-3.29	-9.48	12.43	0.10	-12.33
	Movie	22.85	27.82	4.97	7.68	-14.13	-21.81
2	House	7.94	-4.48	-12.42	6.85	-11.72	-18.58
	Dog	2.68	0.73	-1.95	29.67	-4.15	-33.83
	Movie	20.11	0.09	-20.02	42.39	33.62	-8.77
3	House	10.60	16.30	5.71	-21.80	-15.78	6.02
	Dog	-9.63	11.96	21.60	16.54	-11.72	-28.26
	Movie	23.53	-3.83	-27.36	-8.08	-8.60	-0.52
4	House	-4.20	-10.24	-6.04	18.12	9.70	-8.42
	Dog	2.76	-4.91	-7.66	36.91	-9.48	-46.39
	Movie	0.02	6.87	6.85	29.48	20.57	-8.92
5	House	-12.86	-7.99	4.86	33.88	21.62	-12.27
	Dog	-2.58	-4.93	-2.35	38.58	13.41	-25.17
	Movie	17.21	0.80	-16.41	7.41	-5.77	-13.18
6	House	63.90	7.17	-56.73	22.40	-3.37	-25.76
	Dog	26.82	1.86	-24.96	15.74	6.92	-8.81
	Movie	46.02	-8.65	-54.67	65.72	3.38	-62.34
7	House	11.38	-8.88	-20.26	6.66	-6.76	-13.42
	Dog	15.89	-5.29	-21.19	17.88	-1.89	-19.77
	Movie	30.62	7.12	-23.50	24.56	-11.08	-35.64
8	House	24.21	-2.10	-26.30	52.26	5.42	-46.85
	Dog	77.89	29.35	-48.53	59.67	50.70	-8.97
	Movie	-5.23	-0.44	4.79	43.34	51.55	8.21
9	House	14.37	16.65	2.29	-5.56	-6.24	-0.69
	Dog	9.58	14.98	5.41	5.70	1.34	-4.37
	Movie	-2.46	-8.99	-6.54	6.93	0.02	-6.91
10	House	4.93	-4.60	-9.53	21.42	9.84	-11.58
	Dog	-17.36	-10.99	6.37	10.86	-6.73	-17.59
	Movie	-10.81	-10.89	-0.08	24.76	0.35	-24.40
11	House	-3.95	-5.89	-1.95	3.88	-7.53	-11.41
	Dog	-0.71	2.14	2.85	-13.17	-16.98	-3.81
	Movie	-20.13	0.73	20.85	20.99	23.99	3.00
12	House	-8.35	-4.79	3.56	-0.30	-7.29	-6.99
	Dog	-8.56	-8.77	-0.21	-5.62	0.18	5.79
	Movie	0.72	-1.36	-2.08	-4.94	-6.02	-1.08

(*continued on next page*)

Table E1 (*continued*)

13	House	20.25	1.63	-18.62	54.73	45.42	-9.31
	Dog	10.02	-3.06	-13.08	10.06	-13.41	-23.47
	Movie	37.92	25.91	-12.01	9.20	0.31	-8.89
14	House	54.08	-2.81	-56.89	9.62	-0.71	-10.33
	Dog	0.14	-11.97	-12.10	31.64	23.38	-8.26
	Movie	134.09	8.27	-125.83	64.80	56.56	-8.25
15	House	3.83	5.26	1.43	40.91	6.40	-34.51
	Dog	9.46	17.59	8.13	42.37	33.63	-8.74
	Movie	21.93	14.66	-7.27	19.47	11.26	-8.21
16	House	-6.25	-12.17	-5.92	62.40	52.81	-9.59
	Dog	65.82	11.90	-53.92	66.33	8.22	-58.11
	Movie	16.19	1.39	-14.80	43.84	-2.39	-46.23
17	House	26.16	17.69	-8.47	14.05	5.21	-8.85
	Dog	93.26	82.99	-10.27	6.81	1.45	-5.36
	Movie	38.81	28.03	-10.78	-10.26	-12.35	-2.08
18	House	11.80	14.41	2.61	7.02	-4.31	-11.33
	Dog	3.87	-5.22	-9.09	37.19	19.68	-17.52
	Movie	17.64	11.32	-6.32	58.08	-4.74	-62.82
19	House	-10.01	-12.03	-2.01	22.37	14.66	-7.71
	Dog	21.59	-7.27	-28.85	20.47	12.25	-8.21
	Movie	34.93	1.85	-33.08	29.19	5.80	-23.39
20	House	-5.60	-8.03	-2.43	45.93	37.57	-8.36
	Dog	4.63	9.44	4.82	-0.81	1.91	2.72
	Movie	-14.65	-11.74	2.90	57.92	16.80	-41.12
21	House	10.73	2.88	-7.85	20.69	12.02	-8.67
	Dog	4.58	-8.72	-13.30	37.67	-11.66	-49.34
	Movie	6.25	-5.60	-11.85	-7.97	-12.01	-4.04
22	House	7.10	-0.15	-7.25	40.24	6.89	-33.34
	Dog	26.32	38.05	11.73	-9.32	-10.03	-0.71
	Movie	-0.80	-2.43	-1.63	-2.88	-10.00	-7.12
23	House	-16.81	-19.68	-2.87	25.85	-7.84	-33.70
	Dog	-16.79	-6.41	10.39	28.67	9.22	-19.45
	Movie	-0.93	-9.37	-8.44	-3.34	-0.08	3.27
24	House	-7.28	-10.38	-3.11	0.94	15.18	14.24
	Dog	-2.74	-1.96	0.78	-5.59	-1.46	4.13
	Movie	2.05	-9.55	-11.60	0.46	-8.22	-8.68

(*continued on next page*)

Table E1 (*continued*)

25	House	6.01	-3.60	-9.61	21.82	-2.89	-24.70
	Dog	47.79	-8.31	-56.10	24.64	-8.34	-32.98
	Movie	-3.44	-7.51	-4.06	29.30	0.44	-28.85
26	House	8.67	-7.36	-16.03	21.69	-9.54	-31.24
	Dog	6.92	-4.35	-11.27	69.74	61.24	-8.50
	Movie	19.20	17.05	-2.15	66.83	58.19	-8.64
27	House	24.82	18.15	-6.68	59.25	49.90	-9.35
	Dog	36.62	3.15	-33.47	53.05	44.36	-8.69
	Movie	27.49	0.31	-27.17	68.69	60.06	-8.64
28	House	23.81	15.55	-8.25	31.66	22.88	-8.78
	Dog	66.61	42.04	-24.57	47.72	39.23	-8.49
	Movie	65.66	56.41	-9.25	48.08	38.88	-9.21
29	House	5.37	-3.52	-8.89	-2.16	-8.16	-6.00
	Dog	31.50	24.29	-7.21	8.00	34.07	26.07
	Movie	8.47	-11.58	-20.05	-4.24	9.89	14.13
30	House	38.82	0.69	-38.13	17.83	-10.95	-28.78
	Dog	26.04	15.86	-10.18	10.66	12.60	1.94
	Movie	13.46	-7.25	-20.70	4.19	11.60	7.41
31	House	34.59	18.35	-16.23	7.88	4.18	-3.70
	Dog	-3.76	-4.28	-0.52	7.04	-2.34	-9.37
	Movie	14.28	12.18	-2.10	11.77	-9.13	-20.89
32	House	5.35	-7.78	-13.13	-7.65	-11.16	-3.50
	Dog	4.43	-5.37	-9.80	-13.88	-11.59	2.30
	Movie	-19.96	-12.30	7.67	1.05	-3.43	-4.48
33	House	-4.97	-12.98	-8.01	-15.92	-13.66	2.26
	Dog	-7.17	-2.43	4.74	0.27	-3.83	-4.10
	Movie	-3.78	-2.10	1.68	-0.74	-2.82	-2.08
34	House	5.00	-6.37	-11.37	11.24	-9.09	-20.33
	Dog	-1.17	-4.80	-3.63	79.51	69.03	-10.48
	Movie	3.01	-3.61	-6.62	20.36	-13.21	-33.56
35	House	-0.62	-8.21	-7.59	3.73	-4.58	-8.31
	Dog	3.44	5.93	2.49	8.86	2.14	-6.73
	Movie	4.14	17.98	13.84	75.10	66.27	-8.83
36	House	3.50	7.83	4.33	-7.78	-0.98	6.80
	Dog	18.00	30.09	12.09	-12.78	-13.78	-1.00
	Movie	-0.65	1.99	2.65	-21.36	-11.02	10.34
37	House	-7.72	-5.37	2.36	-6.09	-3.28	2.81
	Dog	-7.19	-8.74	-1.55	-7.80	-13.33	-5.53
	Movie	5.38	4.89	-0.49	4.87	10.49	5.62
38	House	2.93	0.36	-2.57	15.69	-3.23	-18.91
	Dog	7.87	-3.38	-11.24	-1.14	-6.00	-4.86
	Movie	36.86	-14.12	-50.98	37.22	8.02	-29.20

(*continued on next page*)

Table E1 (*continued*)

39	House	2.51	-5.63	-8.14	31.72	3.09	-28.63
	Dog	18.07	12.02	-6.05	2.52	-8.35	-10.87
	Movie	19.12	14.42	-4.69	-1.00	-6.33	-5.33
40	House	10.20	-8.35	-18.54	3.24	17.28	14.05
	Dog	13.30	0.70	-12.61	51.71	19.56	-32.14
	Movie	3.63	-9.82	-13.46	27.19	-10.17	-37.36
41	House	8.30	3.98	-4.32	-2.54	6.27	8.81
	Dog	1.97	-4.80	-6.76	13.64	-4.04	-17.69
	Movie	2.21	0.59	-1.62	15.99	-8.78	-24.77
42	House	10.10	-2.19	-12.29	-5.92	-13.92	-8.01
	Dog	3.72	-5.50	-9.21	-5.42	-9.56	-4.15
	Movie	3.23	7.54	4.31	-6.90	-13.70	-6.80
43	House	2.06	-1.37	-3.43	52.76	25.16	-27.60
	Dog	-12.97	-13.18	-0.21	9.39	2.08	-7.31
	Movie	15.07	-0.49	-15.56	4.92	1.77	-3.15
44	House	-4.91	-11.39	-6.48	4.50	-7.32	-11.83
	Dog	0.17	-6.53	-6.70	17.65	-6.14	-23.79
	Movie	-8.97	-10.88	-1.91	26.22	1.30	-24.92
45	House	8.59	2.11	-6.48	47.77	46.39	-1.38
	Dog	21.33	15.62	-5.71	21.48	12.66	-8.82
	Movie	2.09	-3.19	-5.28	18.87	18.77	-0.10

Appendix F

Fitted and raw data for OQS4

We plot fitted vs. raw (smoothed) data for OQS4 only, as this turned out to be the superior model (Fig. F1). We show the data for the first four participants – the plot for the remaining participants are available in the project OSF page (https://osf.io/vpdx5/).

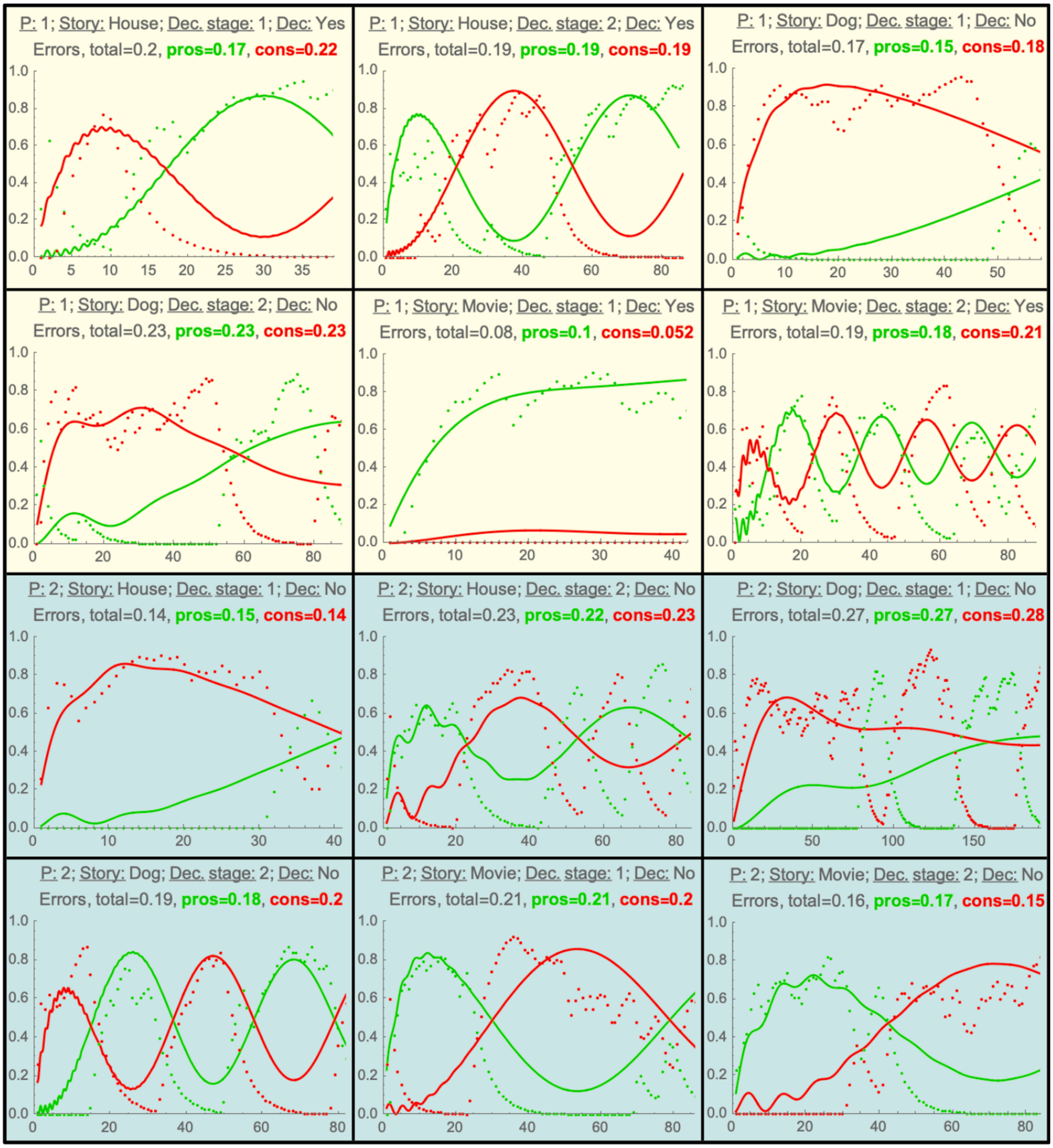

Fig. F1. Fitted vs. raw (smoothed) data for OQS4. Dotted lines represent smoothed raw data and continuous lines best model fits. Green and red lines correspond to data and fits for pros and cons respectively. The Errors (total, pros, cons) are RSS/ (time bins) values.

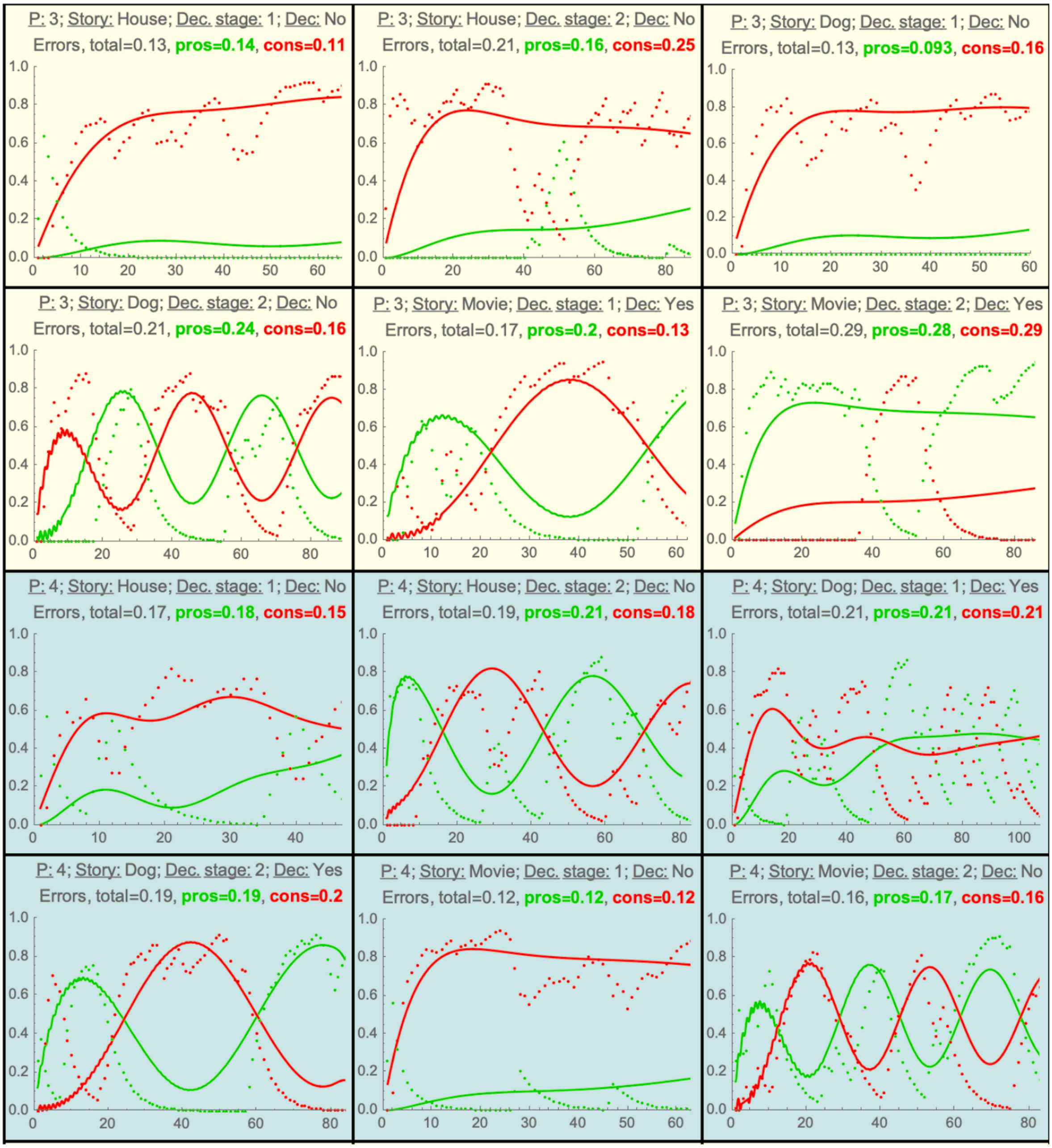

Fig. F1. (*continued*).

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